

Homework 11
Due Wednesday, Nov 12

Explain all your work. Correct answer with no work shown may receive zero credit.

Problem 1. Denote the roots of $f(x) = x^3 + 5x^2 + 3x - 4$ by x_1 , x_2 , and x_3 . Find

- (1) $\sqrt{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}}$
- (2) $x_1^2 + x_2^2 + x_3^2$
- (3) $\frac{x_1}{x_2x_3} + \frac{x_2}{x_1x_3} + \frac{x_3}{x_1x_2}$
- (4) $x_1^3 + x_2^3 + x_3^3$

Problem 2. Denote the roots of $f(x) = 2x^3 + 3x^2 - 5x - 2$ by x_1 , x_2 , and x_3 . Find a polynomial whose roots are

- (1) $3x_1$, $3x_2$, and $3x_3$
- (2) $x_1 + 1$, $x_2 + 1$, and $x_3 + 1$

Problem 3. Factor $f(x) = x^4 - 2x^3 + 3x^2 - 2x + 2$ into irreducible polynomials in

- (a) $\mathbb{R}[x]$
- (b) $\mathbb{C}[x]$

Problem 4. Check that the number $\sqrt{3} - \sqrt{2}$ is a solution of the equation $f(x) = x^4 - 10x^2 + 1 = 0$. Deduce from this that $\sqrt{3} - \sqrt{2}$ is irrational.

Problem 5. Find all the values of the integer parameter a for which the polynomial $f(x) = x^3 - 3x^2 + ax - 1$ has at least one integer root.

Problem 6. Let n be a positive integer. What is the remainder after n^2 is divided by $n + 4$? (For different values of n you may get different remainders.)

Problem 7. One of the two cross-country skiers spent 20 minutes less than the other covering a distance of 30 km. The speed of the first skier was by 3 km/h higher than the speed of the second. Find the speeds of the skiers.