

Homework 12
Due Wed Nov 19

Problem 1. Find the inverse of each element in $U(15)$.

Problem 2. Does \mathbb{R}^2 form a group with respect to coordinate-wise multiplication? Which points do you need to throw away to get a group?

Problem 3. Show that $\Gamma_4 = \{1, -1, i, -i\}$ is a group with respect to multiplication. Construct a multiplication table for this group.

Problem 4. Construct all possible group tables for a group of order 4.

Problem 5. The *stochastic group* $\Sigma(2, \mathbb{R})$ consists of all those matrices in $GL(2, \mathbb{R})$ whose column sums are 1, that is, all matrices

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

with non-zero determinant such that $a + b = c + d = 1$. Prove that the product of two stochastic matrices is again stochastic, and that the inverse of a stochastic matrix is stochastic.

Problem 6. Let

$$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Show that $SO(2, \mathbb{R}) = \{R_\theta \mid \theta \in \mathbb{R}\}$ is a group with respect to matrix multiplication. Don't forget to show that $SO(2, \mathbb{R})$ is closed under the operation.