

Homework 3
Due Wednesday, Feb 18th

Explain your solutions. A correct answer with no work shown will receive a score of zero.

Problem 1. The radii of two concentric circles have a ratio of 1: 2. A chord of the bigger circle is split by the smaller circle in three equal parts. Find the ratio of this chord to the diameter of the smaller circle.

Problem 2. The point P is 7 units away from the center O of a circle whose radius is 11. A chord passing through P has length 18. Find lengths of the segments into which the chord is split by P .

Problem 3. Let $ABCD$ be a square with a side a . A circle inscribed in the square touches the side CD at the point E . Find the length of the chord cut out by the circle from the segment AE .

Problem 4. Let a tangent and a chord be drawn from a point A to a circle. The length of the tangent is 16 while the distance from A to the farther common point of the circle and the chord is 32. Find the radius of the circle, given that the distance from the center of the circle to the chord is 5.

Problem 5. From a point M that lies outside of two concentric circles draw four lines that are tangent to the circles at the points A, B, C, D . Show that the points A, B, C, D, M all lie on the same circle. Hint: If a right angle is subtended by a diameter, its vertex lies on the circle.

Problem 6. Angle bisectors of two angles in a triangle intersect at an angle of 110° . Find the third angle of the triangle.

Problem 7. Does there exist a triangle two of whose angle bisectors are perpendicular to each other?

Problem 8. An outside equilateral triangle ABM is constructed on the side AB of a square $ABCD$. Find the measure of the angle DMC .

Problem 9. Let ABC be a right triangle with legs $AC = 6$ and $BC = 8$. Find the side of a square that is inscribed in the triangle and shares the right angle C with the triangle.

Problem 10. Let the medians AN and BM of a triangle ABC intersect at the point K . Find the area of ABC given that $AN = 6$, $BM = 9$, and $\angle AKB = 30^\circ$.

Hint: Find the area of AKB first. What is the ratio of the areas of triangles AKB and ABC ?

Bonus 1. Let AD, BE, CF be the three angle bisectors in an isosceles triangle ABC (with $AB = AC$). Find BC given that $AC = 1$ and A, D, E, F all belong to the same circle.

Hint: use the property of the angle bisector (hw 2, Problem 3) and the proposition about a tangent and a chord.

E-mail me at soprunova@math.kent.edu if you have any questions.