

Homework 4
Due Wednesday, Feb 25th

Explain your solutions. A correct answer with no work shown will receive a score of zero. The main theme in this homework is trigonometry and the Laws of Sines and Cosines.

Problem 1. ABC is an isosceles triangle with the base AC ; CD is the bisector of $\angle C$; $\angle ADC = 150^\circ$. Find $\angle B$.

Problem 2. A line that goes through the vertex A of a triangle ABC crosses the side BC at the point M . Given that $BM = BA$, $\angle BAM = 35^\circ$, and $\angle CAM = 15^\circ$, find the angles of ABC .

Problem 3. A side in a triangle is 21, and the two other sides are proportional as 3: 8 and form an angle of 60° . Find these sides. (Hint: Law of Cosines)

Problem 4. Let ABC be an isosceles right triangle. Let D be the point on the continuation of the hypotenuse AB through the vertex B such that $BD = BC$. Find the sides of the triangle ADC given that $BC = a$.

Problem 5. In a right triangle ABC the lengths of the legs AC and BC are 15 and 20. On the hypotenuse AB pick a point D so that $BD = 4$. Find CD .

Problem 6. Let ABC be a right isosceles triangle with the hypotenuse AB . We draw a circle so that AC is the diameter of the circle. Let M be the point where the circle intersects AB . Find the distance from the vertex B to the center O of the circle if $BM = \sqrt{2}$.

Problem 7. One of the sides in a triangle is twice as long as another. The angle between these two sides is 60° . Show that this is a right triangle.

Problem 8. Let R be the radius of a circle circumscribed around the triangle ABC . Let c be the side of ABC opposite to the angle $\angle C$, b be the side opposite to $\angle B$ and a opposite to $\angle A$. Show that

$$\sin \angle C = \frac{c}{2R}.$$

Conclude that the area S_{ABC} of ABC satisfies

$$S_{ABC} = \frac{abc}{4R}.$$

Bonus 1. An isosceles triangle with a base a and angle α at the base is inscribed in a circle. Another circle touches both the base of the triangle and the first circle. Find the radius of the second circle.