

## Homework 5 Due Wednesday, March 3rd

Explain your solutions. A correct answer with no work shown will receive a score of zero. Examples from <http://www.tpub.com/math2/16.htm> could be helpful for some of the problems below.

**Problem 1.** Given an ellipse, explain in detail how to construct a line through a given point on the ellipse that is tangent to the ellipse. Show that the line is tangent to the ellipse. (We did this in class.)

**Problem 2.** Write down an equation of the ellipse whose foci are at  $(\pm\sqrt{5}, 0)$  and whose eccentricity is  $\sqrt{5}/3$ .

**Problem 3.** Complete the squares to get an equation of the ellipse in the standard form

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

Find the major and minor semiaxis, eccentricity, and an equation of the directrix.

- (a)  $9x^2 + 4y^2 - 36x - 24y + 36 = 0$
- (b)  $4x^2 + 18y^2 + 60y + 41 = 0$

**Problem 4.** A parabola is the locus of points equidistant from a given point (focus) and a line (directrix). Find an equation of a parabola with the focus  $F(1, 1)$  and the directrix  $x = -1$ .

**Problem 5.** Find the distance between the point  $A(1, 7)$  and the point of intersection of the lines  $x - y - 1 = 0$  and  $x + 3y - 12 = 0$ .

**Problem 6.** Write down an equation of the line that passes through  $M(-3, 2)$  and is parallel to the line  $2x - 3y + 4 = 0$ .

**Problem 7.** A circle centered at  $M(3, 1)$  passes through the origin. Write down an equation of the circle.

**Problem 8.** Given three points  $A(0, 0)$ ,  $B(4, 0)$ , and  $C(0, 6)$ , write down an equation of the circumcircle for the triangle  $ABC$ .

**Problem 9.** Prove that the line  $3x - 4y + 25 = 0$  is tangent to the circle  $x^2 + y^2 = 25$ . Find the point of tangency.

**Bonus 1.** Draw a closed 6-edge polygonal line each of whose edges intersects exactly one other edge.

**Bonus 2.** Find an equation of a parabola with the focus  $F(1, 1)$  and the directrix  $y = -x$ .

E-mail me at [soprunova@math.kent.edu](mailto:soprunova@math.kent.edu) if you have any questions.