Geometry for High School Teachers
Spring 2009

Exam Practice
Do not hand in

Problem 1. Circles with centers O_1, O_2 are tangent to each other at the point K and neither of them is inside the other. Draw a line which tangent to both circles at the points A and B. Let a common tangent at the point K intersect AB at the point M. Show that $\angle O_1MO_2 = pi/2$.

Problem 2. An angle at the vertex A in the triangle ABC is 120°. Draw a circle that is tangent to BC and the continuations of the sides AC and AB. Show that the distance from A to the center of the circle is equal to the perimeter of ABC

Problem 3. Let ABC be a triangle such that two its medians AM and BN are perpendicular. Given that $BC = a$, $AC = b$, and $AB = c$, show that $a^2 + b^2 = 5c^2$.

Problem 4. Use vectors and dot-product to solve previous problem.

Problem 5. $ABCDA'B'C'D'$ is a cube with side 4, M is the center of the face $A'B'C'D'$ and H is a point on AC such that $AH : HC = 1 : 3$.

(1) Find the angle between AC and BC'
(2) Find the angle between HM and CB'.
(3) Find the distance from B to $\pi(HMC)$
(4) Find the distance from B to the line containing AM
(5) Find the distance between the lines containing HM and BD.

Problem 6. Pump up a dodecahedron to get a sphere of radius 1 (strictly speaking, project the dodecahedron onto the sphere from its center, which is right at the center of the dodecahedron). Now, on the sphere we have twelve congruent spherical pentagons. Find the sum of angles for one of them.

Problem 7. Show that there is no polyhedron such that all its faces are quadrilaterals and there are 7 edges meeting at every vertex. Hint: Assume that such a polyhedron exists, find relations on V, E, F and plug them into Euler's Formula to find a contradiction. This should be similar to the soccer ball problem.

E-mail me at soprunova@math.kent.edu if you have any questions.