

Homework 2
Due Monday, Nov 24th

Problem 1. Let p be a prime number, $p > 3$. Show that $24 \mid (p - 1)(p + 1)$.

Problem 2. For which values of p all three numbers p , $2p + 1$, and $4p + 1$ are prime?

Problem 3. Prime numbers have only two positive divisors. Describe all numbers that have three positive divisors.

Problem 4. Both the dividend and divisor are multiplied by 3. What happened to the quotient and the remainder? What if only the dividend is multiplied by 3?

Problem 5. The product of a number by the sum of its digits equals 2008. Find the number.

Problem 6. Find the prime decomposition for $30!$.

Problem 7. Let x be a positive integer. It is known that out of the five statements

$$2x > 70 \quad x < 100 \quad 3x > 19 \quad x \geq 10 \quad x > 5$$

two are true and three are false. Find all possible values of x .

Problem 8. Find all the integer solutions of

$$(2x + y)(5x + 3y) = 7$$

Problem 9. Decipher

$$\begin{array}{r} \\ + \\ \\ \hline \end{array}$$

Here same letters represent same digits and different letters represent different numbers.

Problem 10. Find a two-digit number x if the sum of its digits is three times the sum of the digits of $x + 1$. Show that there are no other two-digit numbers that satisfy this condition.

Problem 11. Find a two-digit number x if the first digit of $2x$ is the first digit of x plus three and the second digit of $2x$ is the second digit of x minus three. Show that there are no other two-digit numbers that satisfy this condition.

Problem 12. Find two consecutive natural numbers such that the sum of digits of each of them is divisible by 125. (Hint: How does the sum of digits change when you add 1 to a number?)