Advanced Problem Solving II
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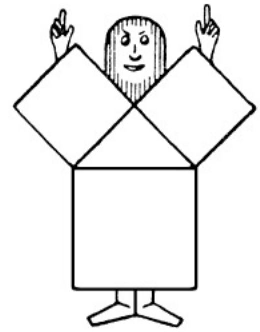
KSU, Fall 2015



Homework 10
Pythagorean Theorem

Prove all your assertions.

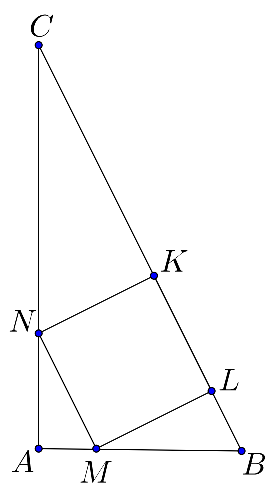
Problem 1. $ABCD$ is a square with side 6. Let O be the center of the squares and let M be a point on the side CD such that $CM : MD = 1 : 2$. Find all the sides of the triangle AOM.

Problem 2. Find the altitude of the trapezoid whose legs (sides) have lengths 6 and 8 and whose bases (the parallel sides) have lengths 4 and 14.

Problem 3. The sides of a triangle have lengths 10, 17, and 21. Find the length of the altitude whose foot is on the largest side.

Problem 4. One base of a right trapezoid is twice as long as the other and the legs (sides) have lengths 4 and 5. Find the lengths of the diagonal of this trapezoid.

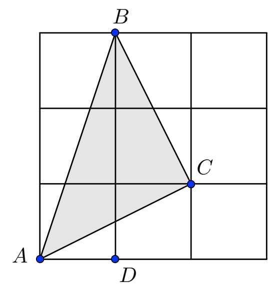
Problem 5. A square $KLMN$ is inscribed into a right triangle ABC so that its side KL is on the hypothenuse BC, as depicted in the diagram below. Given that $CK = a$ and $LB = b$ find the side of the triangle.



Problem 6. A square is inscribed in a right triangle so that the triangle and the square share the right angles and one vertex of the square is on the hypothenuse of the triangle. Given that that vertex breaks the hypothenuse into two segments of lengths a and b, find the side of the square.

Problem 7. Consider a triangle with the vertices $A(0,0)$, $B(1,3)$, and $C(2,1)$ as in the diagram below. Show that ABC is a $45° - 45° - 90°$ triangle. Let D be the point with the coordinates $(1,0)$. Show that $\tan\angle ABD = 1/3$ and $\tan\angle CBD = 1/2$. Explain why this proves the following trig identity:

$$\arctan 1/3 + \arctan 1/2 = \arctan 1.$$



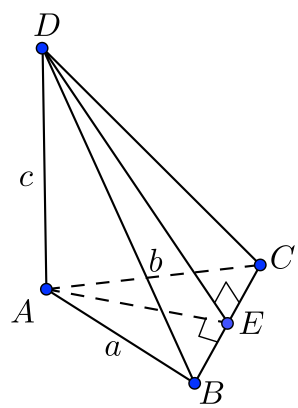
Problem 8. Use a method similar to the one described in the previous problem to show that

$$\arctan\frac{1}{2} = \arctan\frac{1}{3} + \arctan\frac{1}{7}.$$

Problem 9. Use the same method again to show that

$$\arctan 1 + \arctan 2 + \arctan 3 = 180°.$$

Problem 10. Consider a right pyramid $ABCD$ with $\angle BAC = \angle CAD = \angle DAB = 90°$. Show that the sum of squares of the areas of ABC, ABD, and ACD is equal to the square of the area of BCD.



That is, show that

$$S^2_{ABC} + S^2_{ABD} + S^2_{ACD} = S^2_{BCD}.$$

Use the fact that the altitudes from vertices A and D to the side BC have a common foot E. Also, since AD is perpendicular to two lines in base that pass through A, it is perpendicular to every line in the base, in particular $\angle DAE = 90°$.