Advanced Problem Solving II
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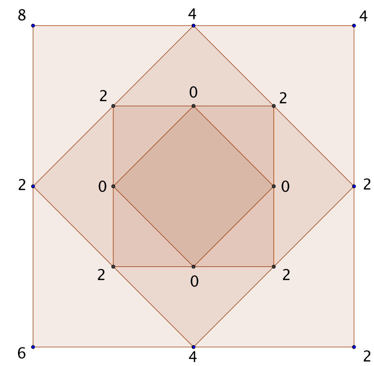

The Four Numbers Game
Homework 1

Problem 1. Play the game

$$(3.2, \pi, \frac{22}{7}, 3.14).$$

Do not use approximations. Do not use any further results.

Problem 2. Show that the possible lengths of 3-Numbers Games, played with non-negative integers, are 0, 1, and ∞. Show that no other lengths are possible and give examples of games of length 0, 1, and ∞. Hint: Go backwards starting with $(0, 0, 0)$.

Problem 3. Show that the games (a, b, c, d) and $(na+e, nb+e, nc+e, nd+e)$ with $n \neq 0$ are equivalent (that is, they end in the same number of steps). Here a, b, c, d, e are non-negative integers and n is a positive integer.

Problem 4. Show that a game (a, b, a, c), where a, b, c are non-negative integers, has a length of at most 4.

Problem 5. Show that a game (a, b, c, d), where $a \geq c \geq b \geq d$ are non-negative integers, has a length of at most 4.

Problem 6. Show that a game (a, b, c, d), where $a \geq b \geq d \geq c$ are non-negative integers, has a length of at most 6.

Problem 7. Consider an 8-Numbers Game (a, b, c, d, e, f, g, h) where all the numbers are integers. Show that all the numbers appearing from step eight onward are even.

Problem 8. Use previous problem to show that every 8-Numbers Game played with non-negative integers has finite length. More precisely, use induction to show that if A is the largest of the eight integers in the beginning of the game and k is the least integer such that $A < 2^k$, then the length of the game is at most $8k$.

Problem 9. Recall that a game (a, b, c, d) is called additive if one of the numbers is equal to the sum of the rest. Construct an additive game S which is equivalent to $(14, 5, 3, 1)$. Next, find a game T that turns into S in one step.

Problem 10. Given an additive game (a, b, c, d) with $a = b+c+d$ construct a game (x, y, z, w) that turns into (a, b, c, d) in one step.

Problem 11. Given a game $S = (a, b, c, d)$ with $a > b + c + d$ construct an additive game equivalent to S.

Problem 12. Show that for any non-negative number N there is an 8-Numbers Game of length N. For this, start with a 4-Numbers Game (a, b, c, d) and construct an 8-Numbers Game based on (a, b, c, d) of the same length. Next, use the corresponding result for 4-Numbers Games discussed in class.

Project Idea For a project, one can study Tribonacci Games, Four Real Numbers Game, Four Numbers game of infinite length, Probability that a four numbers game ends in k steps, k-numbers game. You can also work on the question similar to Problem 2 which refers to 5, 6, or k-Numbers games.