Homework 2
Painting 3D Polyhedra

Problem 1. Some edges of a cube are red and the rest are black. Given that there are no parallel red edges, find the largest possible number of red edges. Explain why this is the largest number of red edges possible under these conditions.

Problem 2. Some edges of a cube are red and the rest are black. Given that there are no red edges in the same face, find the largest possible number of red edges. Explain why this is the largest number of red edges possible under these conditions.

Problem 3. Some edges of a cube are red and the rest are black. Given that there are at most two red edges starting at the same vertex, find the largest possible number of red edges. Explain why this is the largest number of red edges possible under these conditions.

Problem 4. Some edges of an octahedron are red and the rest are black. Given that there are at most two red edges starting at the same vertex, find the largest possible number of red edges. Explain why this is the largest number of red edges possible under these conditions.

Problem 5. Some edges of an octahedron are red and the rest are black. Given that there are at most three red edges starting at the same vertex, find the largest possible number of red edges. Explain why this is the largest number of red edges possible under these conditions.

Problem 6. The edges of an octahedron are painted in three colors: black, red, and blue. Given that at each vertex there are edges of all three colors, find (a) the largest and (b) the smallest number of red edges possible under these conditions. Explain why these numbers are the largest and the smallest.

The pictures below will be useful for solving further problems. The first two are Schlegel diagrams of the icosahedron and dodecahedron: One of the faces was stretched out and the rest was flattened to fit inside this face.
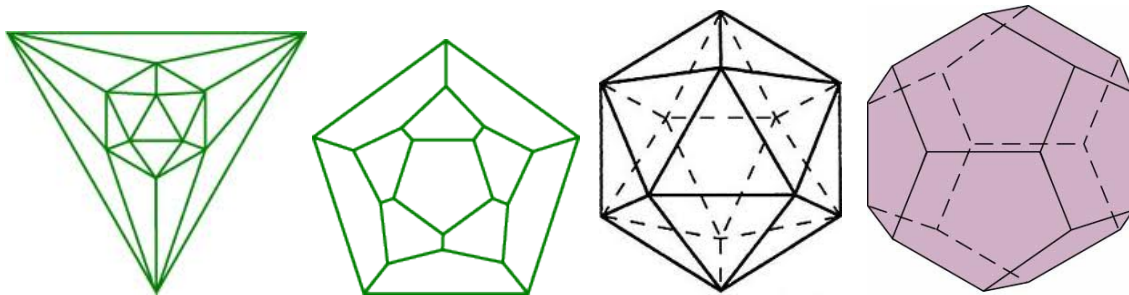
Problem 7. The edges of an icosahedron are painted in three colors: black, red, and blue. Given that at each vertex there are edges of all three colors, find the largest

number of red edges possible under these conditions. Explain why this is the largest number of red edges possible under these conditions.

Problem 8. Two edges of a polytope are *neighbors* if they share a vertex. What is the smallest number of colors one needs to use to paint all the edges of (a) icosahedron (b) dodecahedron so that for each edge all its neighbors are of different colors? Explain why this is smallest number of colors needed under these conditions. (Each edge is colored in one color.)

Problem 9. The faces of an icosahedron are painted in three colors: black, red, and blue. Given that at each vertex we have faces of all three colors, find (a) the largest and (b) the smallest possible number of red faces. Explain why these numbers are the largest and the smallest.

Problem 10. The faces of an icosahedron are painted in four colors: white, black, red, and blue. Given that at each vertex we have faces of all four colors, find (a) the largest and (b) the smallest possible number of red faces. Explain why these numbers are the largest and the smallest.

Problem 11. The faces of a dodecahedron are painted in two colors: black and red. Given that at each vertex we have at most two red faces, find the largest possible number of red faces. Explain why this is the largest number of red faces possible under these conditions.

Problem 12. The faces of a dodecahedron are painted in two colors: black and red. Given that each red face has exactly two red neighbors (two faces are neighbors if they share an edge), find the largest possible number of red faces. Explain why this is the largest number of red faces possible under these conditions.