MATH 64091
KSU, Fall 2016
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Checkerboard Problems

Example 1. Can you tile an 8×8 checkerboard with a corner square cut out with 2×1 domino tiles? What if two opposite squares are cut out?

The first question is easy – the tiling is impossible since each tile covers 2 squares but the overall number of squares is 63, which is odd.

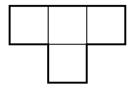
To answer the second question, consider the standard black and white coloring of the checkerboard. Each domino covers one black and one white square. The opposite corner squares are of the same color, so there are 30 squares of one color and 32 of the other. Hence the board cannot be tiled with 2×1 domino tiles.

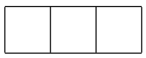
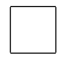
Example 2. Twenty five very jealous people live in twenty five houses that form a five by five square. Each of them thinks that their neighbors (to the North, South, West, and East) have better houses. Can you arrange a move so that each of them moves to one of their neighbor's houses and no two people move to the same house.

No. Let's color the houses red and blue in the checkerboard pattern. Then we will have, say, 12 red houses and 13 blue ones. The owners of red houses want to move to the blue ones, and vice versa, the owners of the blue houses want to move to the red ones. Such a move is impossible since we have different numbers of red and blue houses.

Homework Problems

Problem 1. Can you tile a 10×10 checkerboard with tetrominoes of the shape below.



Problem 2. Can you tile an 8×8 checkerboard with twenty one tiles that look like this:  and one tile that looks like this:  ?

Problem 3. Is it possible to tile a 6 by 10 board with 15 dominoes of size 1 by 4? (Hint: 4-colored board?)

Problem 4. Can a knight travel across an 8×8 checkerboard from the lower left square to the upper right square visiting every of the 64 squares exactly once? (A knight jumps in an L-shape: 1 square up or down and then 2 squares left or right, OR 2 squares up or down and then 1 square left or right. In each move it visits just one new square.)

Problem 5. Can a knight travel across a 3×3 checkerboard visiting every of the 9 squares exactly once? (The knight is jumping, as in the previous problem.) What about 3×4 board?

Problem 6.
- (a) In how many ways can one place eight rooks on an 8×8 checkerboard so that no two rooks attack each other?
- (b) In how many ways can one place four red and four blue rooks on an 8×8 checkerboard so that no two rooks attack each other?
- (c) In how many ways can one place four red and two blue rooks on an 8×8 checkerboard so that no two rooks attack each other? (We say that two rooks attack each other if they are in the same row or column.)

Problem 7. Color some squares of an 8×8 board red so that each square (including the red ones) has exactly one red neighbor. (Each square has four neighbors - two horizontal and two vertical.)

Problem 8. One can tile a 4×4 board with two copies of a 2×4 rectangle. Find five other shapes, each built from eight 1×1 squares, such that two copies of each shape can also tile the 4×4 board. The shapes in this problem are considered to be the same if they are congruent or become congruent after you flip one of them.

Problem 9. Show that it is impossible to cover a 4×5 rectangle with a complete set of tetrominoes, i.e, using each tetromino once.

Problem 10. Use mathematical induction to show that a $2^n \times 2^n$ checkerboard with one square cut out can be tiled by L-trominoes. Warning: It's not given which square is cut out.