MATH 64091
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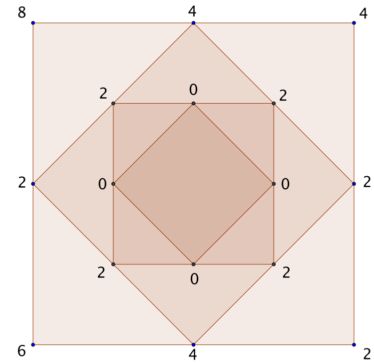

The Four Numbers Game

Problem 1. Play the game
$$(3.2, \pi, \frac{22}{7}, 3.14).$$
Do not use approximations.

Problem 2. Show that the possible lengths of 3-Numbers Games, played with nonnegative integers, are 0, 1, and ∞. (Show that no other lengths are possible and give examples of games of length 0, 1, and ∞).

Problem 3. Show that the games (a, b, c, d) and $(na + e, nb + e, nc + e, nd + e)$ are equivalent (that is, they end in the same number of steps). Here a, b, c, d, e are nonnegative integers and n is a positive integer.

Problem 4. What are the possible lengths of 5-Numbers Games played with nonnegative integers? For each of the possible lengths, give an example. Prove that there are no other possible lengths.

Problem 5. What are the possible lengths of 6-Numbers Games played with nonnegative integers? For each of the possible lengths, give an example. Prove that there are no other possible lengths.

Problem 6. Give another proof that the 4-Numbers Game S played with nonnegative integers has finite length by proving first that if $a > b > c > d$, then the largest integer after two steps is strictly less than the largest integer at the start. Give an example of such a 4-Numbers Game where the largest integer after one step is not less than the largest integer at the start.

Problem 7. Show that a game (a, b, c, d), where $a \geq c \geq b \geq d$ are non-negative integers, has a length of at most 4.

Problem 8. Show that a game (a, b, c, d), where $a \geq b \geq d \geq c$ are non-negative integers, has a length of at most 6.

Problem 9. Show that if the 8-numbers game with nonnegative integer start numbers has length at least 8, then all the numbers appearing from Step 8 onward are even.

Problem 10. Use previous problem to show that every 8-Numbers Game played with nonnegative integers has finite length. More precisely, use induction to show that if A is the largest of the four integers in the beginning of the game and k is the least integer such that $A < 2^k$, then the length of the game is at most $8k$.

Problem 11. Show that for any nonnegative number N there is an 8-Numbers Game of length N. For this, start with a 4-Numbers Game (a, b, c, d) and construct an 8-Numbers Game based on (a, b, c, d) of the same length. Next, use the corresponding result for 4-Numbers Games proved in class.

Project Idea For a project, one can study Tribonacci Games, Four Real Numbers Game, Four Numbers game of infinite length, Probability that a four numbers game ends in k steps, k-numbers game.