

## Kent Regional Algebra Weekend

16–17 April 2011

Schedule

### Saturday 16 April

12:30 -1:15 Sergio Lopez-Permouth, Ohio University  
Measuring modules: alternative perspectives in module theory

1:20 - 2:05 James Wilson, The Ohio State University  
Tools to Tame the Tensor

2:10 - 2:55 Wen Fong Ke, National Cheng Kung University, Taiwan  
Block Intersection Numbers of Certain Block Designs

2:55–3:30 Break

3:30–4:15 Harald Ellers, Allegheny College  
The centralizer of a subgroup in a group algebra

4:20–5:05 Kiumars Kaveh, University of Pittsburgh  
Convex bodies and algebraic varieties

### Sunday 17 April

8:30–9:00 Coffee

9:00–9:45 Edmund Puczyłowski, University of Warsaw  
On the Linear Properties of the Goldie Dimension

9:50–10:35 Silvia Onofrei, The Ohio State University  
Saturated fusion systems with parabolic families

10:35–11:05 Break

11:05 – 11:50 Leonid Makar-Limanov, Wayne State University  
The Freiheitssatz for Poisson Algebras

11:55 – 12:40 Jonathan Hall, Michigan State University  
Triality: algebraic, geometric, and group theoretic

## Abstracts

---

Harald Ellers

Title: The centralizer of a subgroup in a group algebra

Abstract: Let  $F$  be a field,  $G$  a finite group,  $H$  a subgroup of  $G$ . The group algebra  $FG$  is the algebra consisting of all formal linear combinations of elements of  $G$  with coefficients in  $F$ . The centralizer algebra  $FG^H$  is  $\{a \in FG \mid ah = ha \forall h \in H\}$ .

We will discuss two issues.

1) How are simple modules over  $FG^H$  related to simple modules over  $FG$  and  $FH$ ?

2) How is the center of  $FG^H$  related to the centers of  $FG$  and  $FH$ .

We will focus on examples where  $G = S_n$ , the symmetric group, and  $H = S_m$  with  $m < n$ .

---

Jonathan Hall

Title: Triality: algebraic, geometric, and group theoretic

Abstract: Alternative division algebras arose naturally in Cartan's 1925 work on the automorphisms of Lie groups of type  $D_4$  and in Moufang's 1935 work on projective planes satisfying the "Little" Theorem of Desargues. These original examples of triality—in, respectively, algebraic, group theoretic, and geometric contexts—have broad generalizations, which are essentially the same in a categorical setting.

---

Kiumars Kaveh

Title: Convex bodies and algebraic varieties

Abstract: We discuss a new connection between algebraic geometry and convex geometry. We explain a basic construction which associates convex

bodies to semigroups of integral points. We see how this gives rise to convex bodies associated to algebraic varieties encoding information about their geometry. This far generalizes the notion of Newton polytope of a Laurent polynomial/toric variety. As an application, we give a formula for the number of solutions of an algebraic system of equations on any variety, in terms of volumes of these bodies, far generalizing the well-known Bernstein-Kushnirenko theorem. This has many interesting applications in algebraic geometry, in particular theory of linear systems. For the most part, the talk should be accessible to anybody with some background in algebra and geometry. There are many interesting problems in this area yet to be addressed.

---

Wen-Fong Ke

Title: Block Intersection Numbers of Certain Block Designs

Abstract: Let  $(G; +)$  be a finite group of order  $\nu$ , and  $U$  a fixed-point free group of automorphisms of  $G$  with order  $k \geq 2$ . We refer to such a pair  $(G; U)$  a Ferrero pair. If one defines *blocks* as the subsets of  $G$  of the type  $Ua + b = \{u(a) + b \mid u \in U\}$  for  $a, b \in G$  with  $a \neq 0$ , then one obtains a simple  $2$ - $(\nu, k, k - 1)$  design with interesting combinatorial, geometrical, and statistical applications.

Important examples of finite Ferrero pairs are the *field generated* ones. Such one comes from a finite field  $(F, +, \cdot)$  and a nontrivial sub- group  $U_k$  of order  $k$  of the multiplicative group of  $F$ , whose elements are viewed as automorphisms of the additive group  $(F, +)$  via multiplication. In this case, we denote the  $2$ -design obtained by  $\mathcal{B}_{F,k}$  or  $\mathcal{B}_{q,k}$ , where  $q = |F|$  is a power of some prime.

For a block design, one talks about the block intersection numbers. A positive integer  $r$  is said to be *ablock intersection number* of the design if there are two blocks intersecting exactly at  $r$  points.

In this talk, we will discuss the maximal block intersection numbers for eld generated  $2$ -design  $\mathcal{B}_{q,k}$  as described above.

---

Sergio Lopez-Permouth

Title: Measuring modules: alternative perspectives in module theory

Abstract: We will consider various new ways to gauge the projectivity or injectivity of modules. As an illustration of the usefulness of these new approaches, we will focus on modules which are weakest in terms of projectivity or injectivity. We will show how the related notions are interesting in their own right.

---

Leonid Makar-Limanov

Title: The Freiheitssatz for Poisson Algebras

Abstract: In my talk I remind what is the Freiheitssatz type theorem, recall in which situations FT is proved, outline the recent proof (with Umirbaev) of FT for Poisson algebras, and state some open problems related to FT.

---

Silvia Onofrei

Title: Saturated fusion systems with parabolic families

Abstract: In this talk I will elaborate on the connections between fusion systems, chamber systems and parabolic systems.

A fusion system  $\mathcal{F}$  is a category whose objects are the subgroups of a finite  $p$ -group  $S$ ; the morphisms are monomorphisms between these subgroups, such that all monomorphisms induced by conjugation in  $S$  are included. The saturated fusion systems satisfy extra conditions which model properties of the fusion in a finite group related to the Sylow  $p$ -subgroups.

A finite  $p$ -group  $S$  is a Sylow  $p$ -subgroup in a discrete group  $G$  if every finite

$p$ -subgroup of  $G$  is conjugate to a subgroup of  $S$ . I will discuss the relations between the fusion system over  $S$  which is given by conjugation in  $G$  and a certain chamber system  $\mathcal{C}$ , on which  $G$  acts chamber transitively with chamber stabilizer the normalizer of  $S$  in  $G$ .

I will also introduce the notion of a fusion system with a parabolic family and I will show that a chamber system can be associated to such a fusion system. I will give an application to fusion systems with parabolic families of classical type.

---

Edmund Puczyłowski

Title: On the Linear Properties of the Goldie Dimension

Abstract: The Goldie dimension of a module  $M$  is defined as the supremum of all cardinalities  $\lambda$  such that  $M$  contains the direct sum of  $\lambda$  non-zero submodules. This gives a generalization of the linear dimension from linear spaces to modules. The linear dimension can be characterized in several other ways and thanks of that it is so useful tool in many studies. In that context it is natural to ask which (or how far) the fundamental properties of the linear dimension can be extended to the Goldie dimension. Problems of that sort were studied in many papers. The aim of the talk is to present some old and new results concerning that topic.

---

James Wilson

Title: Tools to Tame the Tensor

Abstract: Bilinear maps lurk behind many problems for groups and algebras. They are present when considering the multiplication of a ring or nonassociative algebra, they define the action on a module, the classical groups are described by bilinear forms, and in more subtle ways the commutation in a nilpotent group is encoded by bilinear maps. At first glance, the only tool for

working with bilinear maps is the tensor product, and little else is considered in the world of algebra. In this talk we will introduce various tools with roots in analysis that are ideally suited to answer problems about bilinear maps in general. In effect, there is more than one natural tensor. These techniques answer problems for groups, rings, and algebras that were once thought to be quite difficult. We organize the talk around a tour of these tools. To do this we focus on a case study from group theory: to completely describe the automorphisms of relatively-free groups of an arbitrary group variety satisfying the laws  $[x, y, z]$  and  $x^p$ , though no serious group theory will be necessary to understand the talk.