GERŠGORIN-TYPE EIGENVALUE INCLUSION THEOREMS AND THEIR SHARPNESS

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Dedicated to Professor John Todd, on the occasion of his 90th birthday, May 16, 2001.

Abstract. Here, we investigate the relationships between \( G(A) \), the union of Geršgorin disks, \( K(A) \), the union of Brauer ovals of Cassini, and \( B(A) \), the union of Brualdi lemniscate sets, for eigenvalue inclusions of an \( n \times n \) complex matrix \( A \). If \( \sigma(A) \) denotes the spectrum of \( A \), we show here that

\[
\sigma(A) \subseteq B(A) \subseteq K(A) \subseteq G(A)
\]

is valid for any weakly irreducible \( n \times n \) complex matrix \( A \) with \( n \geq 2 \). Further, it is evident that \( B(A) \) can contain the spectra of related \( n \times n \) matrices. We show here that the spectra of these related matrices can fill out \( B(A) \). Finally, if \( G^R(A) \) denotes the minimal Geršgorin set for \( A \), we show that

\[
G^R(A) \subseteq B(A).
\]

Key words. Geršgorin disks, Brauer ovals of Cassini, Brualdi lemniscate sets, minimal Geršgorin sets.

AMS subject classification. 15A18.