SINGULAR VALUE DECOMPOSITION
NORMALLY ESTIMATED GERŠGORIN SETS*

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Abstract. Let $B \in \mathbb{C}^{N \times N}$ denote a finite-dimensional square complex matrix, and let $V \Sigma W^*$ denote a fixed singular value decomposition (SVD) of $B$. In this note, we follow up work from Smithies and Varga [Linear Algebra Appl., 417 (2006), pp. 370–380], by defining the SV-normal estimator $\epsilon_{V \Sigma W^*}$, (which satisfies $0 \leq \epsilon_{V \Sigma W^*} \leq 1$), and showing how it defines an upper bound on the norm, $\|B^* B – BB^*\|_2$, of the commutant of $B$ and its adjoint, $B^* = B^T$. We also introduce the SV-normally estimated Geršgorin set, $\Gamma_{NSV}(V \Sigma W^*)$, of $B$, defined by this SVD. Like the Geršgorin set for $B$, the set $\Gamma_{NSV}(V \Sigma W^*)$ is a union of $N$ closed discs which contains the eigenvalues of $B$. When $\epsilon_{V \Sigma W^*}$ is zero, $\Gamma_{NSV}(V \Sigma W^*)$ is exactly the set of eigenvalues of $B$; when $\epsilon_{V \Sigma W^*}$ is small, the set $\Gamma_{NSV}(V \Sigma W^*)$ provides a good estimate of the spectrum of $B$. We end this note by expanding on an example from Smithies and Varga [Linear Algebra Appl., 417 (2006), pp. 370–380], and giving some examples which were generated using Matlab of the sets $\Gamma_{NSV}(V \Sigma W^*)$ and $\Gamma_{RNSV}(V \Sigma W^*)$, the reduced SV-normally estimated Geršgorin set.

Key words. Geršgorin type sets, normal matrices, eigenvalue estimates

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