SINGULAR VALUE DECOMPOSITION NORMALLY ESTIMATED GERŠGORIN SETS*

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Abstract. Let $B \in \mathbb{C}^{N\times N}$ denote a finite-dimensional square complex matrix, and let $V\Sigma W^*$ denote a fixed singular value decomposition (SVD) of B. In this note, we follow up work from Smithies and Varga [Linear Algebra Appl., 417 (2006), pp. 370–380], by defining the SV-normal estimator $\epsilon_{V\Sigma W^*}$, (which satisfies $0 \leq \epsilon_{V\Sigma W^*} \leq 1$), and showing how it defines an upper bound on the norm, $||B^*B - BB^*||_2$, of the commutant of B and its adjoint, $B^* = \bar{B}^T$. We also introduce the SV-normally estimated Geršgorin set, $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$, of B, defined by this SVD. Like the Geršgorin set for B, the set $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$ is a union of N closed discs which contains the eigenvalues of B. When $\epsilon_{V\Sigma W^*}$ is zero, $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$ is exactly the set of eigenvalues of B; when $\epsilon_{V\Sigma W^*}$ is small, the set $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$ provides a good estimate of the spectrum of B. We end this note by expanding on an example from Smithies and Varga [Linear Algebra Appl., 417 (2006), pp. 370–380], and giving some examples which were generated using Matlab of the sets $\Gamma^{\mathrm{NSV}}(V\Sigma W^*)$ and $\Gamma^{\mathrm{RNSV}}(V\Sigma W^*)$, the reduced SV-normally estimated Geršgorin set.

Key words. Geršgorin type sets, normal matrices, eigenvalue estimates

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