1962
Reprinted from Pacific Journal of Mathematics, Vol. 12, No. 4

David G. Feingold and Richard S. Varga

Circle Theorem
Generalizations of the Gershgorin Block Diagonally Dominant Matrices and
For the definition of an $A$-matrix, see § 4 or § 8.

Received April 11, 1962.

This subspace the vector norm $\|x\|$ into itself, we associate with the $n$-dimensional vector subspace $\mathcal{V}$ of the linear transformation $\mathcal{V}$ as a linear transformation

$$[\mathcal{V}] = \mathcal{V}$$

(2.1)

For reasons to appear in § 3, the partitioned choice $\mathcal{V} = \mathcal{V}$ of where the diagonal summatiess $\mathcal{V}$ are square of order $n$, where

$$\begin{pmatrix} \mathcal{V} & \cdots & \mathcal{V} \\
\cdots & \ddots & \cdots \\
\mathcal{V} & \cdots & \mathcal{V} \end{pmatrix} = \mathcal{V}$$

(2.2)

I. Block diagonally dominant matrices. Let $A$ be any $n \times n$ matrix.

2. Block diagonally dominant matrices. Let $A$ be any $n \times n$ matrix.

Cesàro-Horni-Madeira's work on providing bounds for the eigenvalues of $A$.

Exclusion regions can give significant improvements over the usual diagonalization process. But one of our major results (§ 3) is that these new diagonalization regions of the simple concept of a diagonally dominant matrix $A$. By the generalization of the simple concept of a diagonally dominant matrix $A$, through not fundamentally different, establish the non-diagonality of $A$. Through comparisons with $A$-matrices, our approach, nonsingularly of $A$, through comparisons with $A$-matrices, our approach.

Evidently, confirms no eigenvalues of $A$. In a like manner, the following results:

$$I \succ ||A - (I - I^2)||$$

Non-singular. Hence, the set of all complex numbers $z$ for which $A$ is a non-singular diagonal matrix, then Hausdorff distance is $z$. Such exclusion regions arise naturally from results which establish the non-singular of $A$. For example, if $A + D = \mathcal{V}$, where $\mathcal{V}$ is a non-singular diagonal matrix, then Hausdorff distance is $z$. Such exclusion regions for the eigenvalues of an arbitrary square matrix $A$.

I. Introduction. The main purpose of this paper is to give generalizations of the well-known theorem of Cesàro-Madeira's work on providing bounds for the eigenvalues of $A$.

David C. Fringado and Richard S. Yang.

Circle Theorem

Generalizations of the Cesàro-Madeira Block Diagonally Dominant Matrices and
\[ a / I (a I x) \equiv \| x \|^d \text{ to denote the } l^d \text{ norm of } x \]

Later, we shall use the notation \(| x |^{f + \eta} \]

Consider the case \( n = 1 \) of \( f \) and \( \eta \).

An example of a matrix which is block strictly diagonally dominant, which is the usual definition of diagonal dominance.

\[ N \triangleq f \triangleq I \text{ for all } \| x \|_{\Lambda} \leq \| (f + \eta) x \|_{\Lambda} \]

Whenever \((f + \eta) x \) is nonsingular. The inequality \((f + \eta) x \) is singular.

In the special case that all the matrices are \( f \times f \) matrices and \( f \) is block diagonally dominant, relative to the partitioning \((1) \) then \( N \triangleq f \triangleq I \text{ for all } \| x \|_{\Lambda} \leq \| (f + \eta) x \|_{\Lambda} \)

By continuity, to be zero whenever \((f + \eta) x \) is singular.

The case \((f + \eta) x \) can also be characterized form \((1) \).

If we wish to point out the quantity appearing on the right

\begin{equation}
\left( \frac{\| x \|}{\| x^{f + \eta} \|} \right)_{\text{in} x} \leq \| f \| \quad \text{for all } \| x \| \leq \| (f + \eta) x \| \)
\end{equation}

If the diagonal submatrices \((f + \eta) x \) are nonsingular, and \( f \) is also nonsingular, then we shall drop the subscripts on the different vector norms.

If the diagonal submatrices \((f + \eta) x \) are 

\begin{equation}
\left( \frac{\| x \|}{\| x^{f + \eta} \|} \right)_{\text{in} x} \leq \| f \| \quad \text{for all } \| x \| \leq \| (f + \eta) x \| \)
\end{equation}

The point here is that we can associate different vector norms with \( f \) and \( \eta \).

Then, if \( \| x \| \) is defined as usual by \( f \) and \( \eta \) for any \( f \) and \( \eta \), the dual definition states that the column vectors are different subspaces \( f \) and \( \eta \). Now, similarly considering the rectangular matrix

\begin{equation}
N \triangleq f \triangleq I \quad \| f \| \| x \| + \| f + x \| \| \quad \text{for any } \| x \| \leq \| f + x \|
\end{equation}

Theorem 4.
where \( \lambda, \gamma \) are square nonnull submatrices. For strongly connected directed graphs,

\[
p = \begin{bmatrix} \gamma^T & \lambda \end{bmatrix}
\]

Therefore, the results no permutation matrices \( P \in \mathbb{R}^{N \times N} \) such that \( P \) preserves the block diagonal structure of the matrix \( M \).

Since \( M \) is a nonzero vector, normalize \( \| M \| = \frac{1}{\kappa} \) so that \( M \) is a normalized vector.

\[
\| M \| = \frac{1}{\kappa} \text{ and } \| M \| = \frac{1}{\kappa} \text{ for all } i, j \in [N]
\]

Thus, \( \| M \| = \frac{1}{\kappa} \) or \( \| M \| = \frac{1}{\kappa} \).

But this is equivalent to

\[
\| M \| = \frac{1}{\kappa} \text{ or } \| M \| = \frac{1}{\kappa}
\]

Theorem 2. The extension to the case where \( M \) is block diagonal is also block diagonal.

Proof. The extension to the case where \( M \) is block diagonal is also block diagonal.

**Theorem 1.** If the partitioned matrix \( A = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \) is block diagonal, then \( A \) is structurally dominant.

**Definition 2.** The partitioned matrix \( A = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \) is block diagonal if the matrix \( A = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \) is block diagonal.

Obviously, \( A \) is structurally dominant in the sense of Definition 2.
Definition 8. For the partitioned \( n \times n \) matrix \( A \) of (2.1), let the

\[ N \subseteq f \subseteq I \]

satisfy (2.10) for at least one \( f \), I. We say that the diagonal entries of an arbitrary \( n \times n \) complex matrix \( A \) necessarily

reduce to the well known (zero) diagonal matrices. In Theorem 2, we shall

show that if \( f \) is such that all

\[ \sum_{k=1}^{n} \frac{1}{|\alpha_k|} \supseteq 1 - (|I - f A|) \quad (2.10) \]

We again remark that if the partitioning of (2.1) is such that all

\[ \sum_{k=1}^{n} \frac{1}{|\alpha_k|} \supseteq 1 - (|I - f A|) \quad (2.10) \]

Theorem 2. For the partitioned \( n \times n \) matrix \( A \) of (2.1), each eigenvalue

of \( A \) satisfies

\[ \lambda \in \mathbb{C} \quad (2.10) \]

dominant, which gives us

\[ \lambda \in \mathbb{C} \quad (2.10) \]

is an eigenvalue of \( A \), then \( A \) cannot be block strictly diagonally

Thus, we have from Theorem I that \( A \) is non-singular. Hence, if

\[ N \subseteq f \subseteq I \]

for all \( f \), then

\[ \sum_{k=1}^{n} \frac{1}{|\alpha_k|} \supseteq 1 - (|I - f A|) \quad (2.10) \]

Identity matrix, suppose that

\[ I \quad (2.10) \]

If \( I \) is the partitioning of (2.1), and \( I \) is the block

in the introduction, the above theorem leads naturally to

\[ I \quad (2.10) \]

Theorem I here is a special case of a more general result by Ostrowski

Theorem I here is a special case of a more general result by Ostrowski.

\[ I \quad (2.10) \]

is the block

and for the block

\[ I \quad (2.10) \]

of (2.1) and for the block

\[ I \quad (2.10) \]

to this result in the case that all the matrices

Theorem I here is a special case of a more general result by Ostrowski.

Actually, we can regard Theorem I as block diagonally dominant case.

completes the proof for the block strictly diagonally dominant, which

\[ Z = \left[ \begin{array}{cc} a & b \\ c & d \end{array} \right] \]

but as \( a \) is non-singular by hypothesis, then putting

\[ \sum_{k=1}^{n} \frac{1}{|\alpha_k|} \supseteq 1 - (|I - f A|) \quad (2.10) \]

(2.10)

David C. Felingold and Richard S. Varca
Another familiar result of Gershgorin can also be generalized. The proof, depending on a continuity argument, follows that given in [13], p. 287.

To complete the proof of Theorem 1, a similar argument can be applied. A similar argument can be repeated with a repeated by \( f \). Moreover, if \( f \) is a bounded point of \( G \), then

\[
\left\| f^* V \right\|_N \leq \left\| M \right\| \left\| f^* V \right\|_N \leq \left\| I - (1 + I z - f^* V) \right\|
\]

(3.8)

If \( f \), then as before, since \( f \) is an eigenvalue of \( V \), then

\[
M f = f^* V \quad f \in \mathfrak{N}
\]

and

\[
M f = f^* V \quad f \in \mathfrak{N}
\]

Since \( f \) is an eigenvalue of \( V \), then

\[
M f = f^* V \quad f \in \mathfrak{N}
\]

Theorem 2. Let the partitioned matrix \( V \) of (2.1) be block irreducible.

This result of Theorem 2, which is similar to the above of a well-known result, is block irreducible. Which is the analogue of a well-known result of this argument. In this case, Theorem 2 gives exact information about the eigenvalues of \( V \).

[II] Theorem 2

\[ M f = f^* V \quad f \in \mathfrak{N} \]

(2.9)

Thus, we can speak of the boundary of \( G \), as well as the boundary of

\[
\bigcap_{N} G = G
\]

Thus, from (2.9), we conclude that the Gershgorin set \( G \) is closed and bounded. Hence, so

\[
\mathfrak{N} \subseteq f \subseteq \mathfrak{N}
\]

(1.8)

Gershgorin set \( G \) be the set of all complex numbers \( z \) such that
Since $G = G'$ as shown in the figure below, the usual (Gerschgorin) circles, so that $G'$ is the union of two disjoint circles, the same is true for $G$.

By definition, the set $G'$ then consists of the points $z$ for which

$$
I_{\mathbb{C}} \subseteq |z - \bar{z}| \quad I_{\mathbb{C}} \subseteq |z - 6|
$$

That is, $G' = \{(z - \bar{z}, |z - 6|) : \min_{\mathbb{C}} = \frac{1}{\bar{z} - (1 - i')\bar{z}} \}$

That is, $I_{\mathbb{C}} = 1$. On the other hand, direct computation shows that the vector norm $||x||_{\mathbb{C}}$ is apparent implying now the vector norm $x$.

$$
\begin{bmatrix}
\bar{v} & \bar{w} \\
\bar{w} & \bar{z} \\
\bar{v} & \bar{z}
\end{bmatrix}
\begin{bmatrix}
v \\
w \\
v
\end{bmatrix}
= v
$$

Illustration, consider the partitioned matrix from the generalized form of Gerschgorin's Theorem. To give another example of the matrix of $G'$, the eigenvalues of $G'$.

$$
H \text{ is diagonal, then } \text{the remaining } N - m \text{ Gerschgorin sets for the}
$$

David G. Feingold and Richard S. Varca
Let $n$ be a positive integer and define $A$ and $B$ as in Theorem 1. Let $n \times n$ complex matrices $A$ and $B$ be partitioned as in (2.1).

Result by Ostrowski [9].

In an attempt to characterize completely the eigenvalues of a generalized matrix $A$ under the condition that the partitioning of $A$ is such that $A$ is block diagonal, we present the following main theorem. As results in the literature suggest, both $A$ and $B$ have been replaced by $A^T$.

Let $x$ be a boundary point of each of the point sets $C$ such that $x \in C \cap B$. Define $N \equiv \sum_{\gamma \neq \gamma'} \sum_{\gamma' \neq \gamma} \sum_{\gamma'' \neq \gamma'} \sum_{\gamma''' \neq \gamma''} \sum_{\gamma'''} \sum_{\gamma''''} \sum_{\gamma'''''}$ where $I \equiv \sum_{\gamma \neq \gamma'} \sum_{\gamma' \neq \gamma} \sum_{\gamma'' \neq \gamma'} \sum_{\gamma''' \neq \gamma''} \sum_{\gamma'''} \sum_{\gamma'''''}$.

Moreover, if $f$ is block irreducible, and (3.5) holds for some $\epsilon > 0$ and $\delta > 0$, then $N \equiv \sum_{\gamma \neq \gamma'} \sum_{\gamma' \neq \gamma} \sum_{\gamma'' \neq \gamma'} \sum_{\gamma''' \neq \gamma''} \sum_{\gamma'''} \sum_{\gamma'''''}$.

There, all the eigenvalues of $A$ in the union of the $C$-faces of $A$ where $C$ is the union of all $C$-faces of $A$ are partitioned as in (2.1).

Theorem 6. Let $n \times n$ complex matrices $A$ and $B$ be partitioned as in (2.1).

Carothers, O. An example of a $C$-face on a complex $n \times n$ matrix.

Proof. Let the eigenvalues of $A$ be of $\epsilon > 0 > \delta > 0$. Since $A$ is $\epsilon > 0 > \delta > 0$ and $B$ is $\epsilon > 0 > \delta > 0$, we can write, which, combined with the final result, we have that the union of all $C$-faces of $A$ where $C$ is the union of all $C$-faces of $A$ are partitioned as in (2.1).

Thus, a special case of the result of Carothers is considered in this paper. This is a significant improvement over the usual Carothers circle in providing bounds for the eigenvalues of $A$ in the union of all $C$-faces of $A$ where $C$ is the union of all $C$-faces of $A$ are partitioned as in this case.

\[ \| A \| \leq \epsilon \leq \delta \leq 1 \leq \gamma \]

This is a circle of radius $\epsilon$ with center at $\epsilon = 1$. The previous example was such that the final result gives the circle $\epsilon = 1$. From this figure, we can conclude that the block Carothers circle in the plane.

For the matrix $A$ of (2.4) are all given by the single circle $\epsilon = 1$.
\[ \| x \|_2 = \| x \| \] (2.3)

If \( x \) denote the vector with components \( |x_i| \) let \( |x| \) be a column vector with complex components which depends upon the use of complex norms. By this, we mean based on our previous results, we now give a generalization of this result

\[ u \leq f \leq I \quad 0 < \| H \| \]

(4.1)

Another generalization. A further result, due to Tranusk 

partitions, and the purpose can be used to advantage.

We wish to emphasize that, unlike the cases we treated here,

\[ N \leq f \leq I \]

(6.3)

relations:

whenever one occurs on the left-hand side, we agree to put 0/0. Theorem 8. Let the \( n \) complex matrices \( V \) be partitioned as in

\[ f \leq I \quad 0 < \frac{f}{I} \quad \text{and} \quad \frac{f}{f} \leq \frac{f}{I} \]

(2.1)

Theorem 8. Let the \( n \) complex matrices \( V \) be partitioned as in

\[ N \leq f \leq I \]

(3.8)

Theorem 8. Let the \( n \) complex matrices \( V \) be partitioned as in

\[ N \leq f \leq I \]

(3.6)

David C. Pincus and Richard S. Varca
PHILOLOGICAL

If \( \lambda > 0 \), then the determinant of \( \mathbf{A} \) is not an eigenvalue of \( \mathbf{A} \), which completes the proof.

In other words, for any \( z \) with \( \Re z > 0 \), then the matrix \( \Gamma - z \mathbf{I} \) is continuous

\[
\lim_{t \to 0}(\|e^{t\mathbf{A}}\|) = \lim_{t \to 0}(\|e^{t(\Gamma - z \mathbf{I})}\|)
\]

so that from (2.3)

\[
\|\mathbf{x}\| \|e^{t\mathbf{A}}\| \leq \|\mathbf{x}\| \|e^{t(\Gamma - z \mathbf{I})}\|
\]

Next, with (4.4) and the assumption of absolute norms, it follows from

\[
\|\mathbf{x}\| \|e^{t\mathbf{A}}\| \leq |(2)^{1/4}|
\]

and that (2.4) and (2.4)

From the assumption that \( \mathbf{A} \) is an \( M \)-matrix, that \( (\mathbf{A}) \) follows from (2.8) = \( 1 - (\Gamma - z \mathbf{I}) \) and (2.8) = \( \mathbf{A} \) with \( \Re z > 0 \). If \( \mathbf{A} \) strictly diagonally dominant, let \( z \) be any complex number.

For simplicity, we shall consider again only the case where

\[
\Re \lambda < 0
\]

then \( \lambda \) is any eigenvalue of \( \mathbf{A} \), and the vector norms for each subspace \( \mathcal{N} \) are absolute norms.

Furthermore, assume that each submatrix \( \mathbf{A} \) is an \( M \)-matrix, for at least one \( i \), and block diagonally dominant with strict inequalities in (2.4) and let \( i \) be block strictly diagonally dominant (or block irreducible) matrix \( \mathbf{A} \) be partitioned as in

\[
[\mathbf{A}]
\]

then \( \mathbf{A} \) is said to be an \( M \)-matrix.

Next, if \( \mathbf{A} \) is a real \( n \times n \) matrix with \( \Re \mathbf{A} = \mathbf{A} \) and if is nonsingular with \( \mathbf{A} \) in exactly one component of \( \mathbf{A} \) and if is nonsingular with \( \mathbf{A} \) for all \( \Re \lambda < 0 \), then

\[
|\mathbf{x}| = |\mathbf{A}| \mathbf{x} \leq |\mathbf{A}| |\mathbf{x}|
\]

to the property that if \( \mathbf{I} \)

13. Some upper conceraining bounds for eigenvalues of 
finite matrices, Survey of


10. A. M. Ostrowski, On some results of properties of operator matrices and matrices


