

**MATH 4/51001**  
**EXAM I**

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NAME: \_\_\_\_\_

Write clearly and show all work. Put your name on each page. Each problem is worth 20 points. Time allowed is 50 minutes.

1. Using complete sentences, define the following terms.

(a) Intersection (of two sets)

(b) Prime Number

(c) Congruent modulo  $n$

(d) Greatest Common Divisor

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2. Find the Greatest Common Divisor  $d$  of 396 and 312, and find integers  $m$  and  $n$  such that  $d = 396m + 312n$ .

3. Use induction to show that  $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$  for all  $n \geq 1$ .

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4. (a) Find the *additive* inverse of  $[7]$  in  $\mathbb{Z}_{25}$ . Write your answer in the form  $[r]$ , with  $0 \leq r \leq 24$ .

(b) Find the *additive* order of  $[9]$  in  $\mathbb{Z}_{12}$ .

(c) Find the *multiplicative* inverse of  $[14]$  in  $\mathbb{Z}_{33}^\times$ .

(d) Find the *multiplicative* order of  $[4]$  in  $\mathbb{Z}_{13}^\times$ .

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5. Let  $s$ ,  $t$ , and  $n$  be positive integers and let  $d = (s, t)$ .
- (a) Show that  $nd \mid ns$  and  $nd \mid nt$ .
  - (b) Show that if  $c \mid ns$  and  $c \mid nt$ , then  $c \mid nd$ .
  - (c) Explain why (a) and (b) imply that  $(ns, nt) = n(s, t)$ .