MATH 4/51001
EXAM II

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October 25, 2006 NAME:____________________________

Write clearly and show all work. Put your name on each page. Each problem is worth 20 points. Time allowed is 50 minutes.

1. Using complete sentences, define the following terms.

   (a) Inverse Function

   (b) Equivalence Class

   (c) Order of a Group Element

   (d) Center of a Group

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2. In S_7, let $\sigma = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 4 & 7 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 4 & 2 & 5 \end{pmatrix}\begin{pmatrix} 3 & 6 \end{pmatrix}$.

(a) Write $\sigma\tau$ and $\tau\sigma$ as products of disjoint cycles.

(b) Find σ^{-1} and τ^{-1}.

(c) Determine the order of σ and the order of τ.

(d) Determine whether each of σ, τ is an even permutation or an odd permutation. Explain.

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3. Let G be a group and fix an element g in G. Let $\varphi : G \to G$ be the function defined by $\varphi(x) = xg$ for all $x \in G$. Show that φ is a bijection.

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4. Let G be a group and let H be a subgroup of G.
Define a relation $\sim$ on G by $a \sim b$ if and only if $a^{-1}b \in H$. Show that $\sim$ is an equivalence relation.

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5. Let $G = \mathrm{GL}_2(\mathbb{R})$ be the group of 2×2 matrices with real number entries and nonzero determinant, under matrix multiplication. Let $H = \{X \in G \mid \det X > 0\}$ be the set of matrices in G with *positive* determinant. Show that H is a subgroup of G.