

MATH 4/51001  
Homework #11  
**Due: Friday, December 8**

**Reading:**

For Monday, December 4: §7.2

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**Problems to turn in:**

**§3.8:** 13, 14, 22, 25.

- I. Let  $G$  and  $H$  be finite groups such that  $|G|$  and  $|H|$  are relatively prime. Show that if  $\varphi : G \rightarrow H$  is a homomorphism, then  $\varphi$  is the trivial homomorphism (i.e.,  $\varphi(g) = e_H$  for all  $g \in G$ ).
- II. Let  $G_1$  and  $G_2$  be groups and  $G = G_1 \times G_2$ . Let  $N = \{(a, e_2) \in G \mid a \in G_1\}$ , where  $e_2$  denotes the identity element of  $G_2$ . Show that  $G/N \cong G_2$ .
- III. Let  $G$  be an abelian group and let

$$S = \{y \in G \mid y = x^2 \text{ for some } x \in G\},$$
$$T = \{a \in G \mid a^2 = e\}.$$

Show that  $G/T \cong S$ .

- IV. Let  $G$  be a group with a normal subgroup  $N$  of order 5, such that  $G/N \cong S_3$ . Show that  $|G| = 30$ ,  $G$  has a normal subgroup of order 15, and  $G$  has 3 subgroups of order 10 that are not normal.

**Notes:**

- Hint for §3.8 #14: Show that if  $G/N = \langle gN \rangle$ , then every element of  $G$  is of the form  $g^m z$ , where  $z \in N \leq Z(G)$ . Show that all such elements must commute.
  - Hint for §3.8 #25: Look in  $D_4$  for an example.
  - Hint for #I: Use the fact that the image of  $\varphi$  is a subgroup of  $H$ , along with the First Isomorphism Theorem (3.8.9) and Lagrange's Theorem.
  - Hint for #II and #III: Use the First Isomorphism Theorem (3.8.9) with an appropriate homomorphism.
  - On #II, note that  $N \cong G_1$  by Exam III #4.
  - Hint for #IV: Use the Correspondence Theorem (3.8.7(b)) and the subgroups of  $S_3$  (Figure 3.6.5 on Page 148).
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**Problems to be aware of:**

§3.8: **10, 11, 20, 21.**

[Problems in **bold** contain results we will assume and use in the course.]