Math 4/51001
Exam III Review

Exam III will be given in class on **Wednesday, November 22, 2006**.

It will cover Sections 3.3 – 3.7, Homeworks 7 – 9, and material from class October 16 through November 8.

Topics to be covered include:

- **Group Constructions** – direct products, orders of elements in direct products, products of subgroups (including order, conditions when product is a subgroup), General Linear Groups over $\mathbb{Z}_p$, groups of order $\leqslant 8$ (including Q_8 and D_4)
- **Group Isomorphisms** – definition, relation between group tables of isomorphic groups, basic properties of isomorphisms, isomorphism as an equivalence relation, group properties preserved by isomorphism (e.g., orders of elements and subgroups, centers, abelian, cyclic)
- **Cyclic Groups** – isomorphisms between cyclic groups, subgroups of cyclic groups are cyclic, order of a^m in terms of order of a, generators of cyclic groups, classification of all subgroups of a cyclic group (finite or infinite)
- **Permutation Groups** – definition, Cayley's Theorem (in particular, the representation of a finite group as a subgroup of S_n), even permutations and alternating groups, dihedral groups D_n (geometric, permutation group, and abstract descriptions), D_4 in particular
- **Homomorphisms** – definition, examples and non-examples, basic properties (contrast with *isomorphism* properties), image and kernel, images and preimages of subgroups are subgroups, condition on the kernel to guarantee one-to-one, normality of kernel

Topics NOT covered:

- General fields other than $\mathbb{Z}_p$ (Definition 3.3.5)
- Subgroups generated by sets (3.3.8, 3.3.9)
- Exponent of a group (3.5.7, 3.5.9)
- Parity of a permutation related to the polynomial Δ_n (3.6.6)
- Factor sets and factor groups (middle of page 158 to page 162)

Possible types of questions:

- Statements of definitions and/or theorems, taken from the following:

Direct Product	Cayley's Theorem
Klein 4-Group	Alternating Group on n letters
Quaternion Group	Dihedral Group of order $2n$
Product of Subgroups	Group Homomorphism
Order of Product of Subgroups	Trivial Homomorphism
Group Isomorphism	Image of a Homomorphism
Isomorphic Groups	Kernel of a Homomorphism
Permutation Group	Normal Subgroup

- Computational problems:
  - Compute products of subgroups and their orders
  - Find cyclic subgroup generated by an element
  - Compute orders of elements in direct products
  - Show groups isomorphic using group tables
  - Determine if a map is an isomorphism
  - Determine if a map is a homomorphism
  - Compute orders of powers a^m given $o(a)$
  - Find all generators of a cyclic group $\langle a \rangle$ given the order of $\langle a \rangle$
  - Calculate products, orders, centralizers in D_n in terms of permutations and using abstract description
  - Represent a finite group as a subgroup of S_n using (proof of) Cayley's Theorem
  - Compute kernel and image of a given homomorphism
- Proofs similar to those in homework, including:
  - basic properties of direct products
  - isomorphisms between groups
  - showing groups are *not* isomorphic
  - properties of isomorphisms
  - properties of cyclic groups and their subgroups
  - properties of orders of elements and their powers
  - permutation groups and Cayley's Theorem
  - properties of dihedral groups and alternating groups
  - showing a map is or is not a homomorphism
  - properties of homomorphisms
  - properties of image and kernel of a homomorphism
  - images and preimages of subgroups