MATH 4/51002
EXAM II

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March 16, 2007 NAME:______________________________

Write clearly and show all work. Put your name on each page. Each problem is worth 20 points. Time allowed is 50 minutes.

1. Using complete sentences, define the following terms.

   (a) Characteristic of a Ring

   (b) Zero Divisor

   (c) Prime Ideal

   (d) Maximal Ideal

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2. An element a of a ring is called an *idempotent* if $a^2 = a$.

   (a) Show that every element of the ring $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ is an idempotent.

   (b) Show that if D is an integral domain, then D has exactly 2 idempotents.

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3. Show that $I = \{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \in \mathbb{Z}[x] \mid a_0 \text{ is even}\}$ is an ideal of the ring $\mathbb{Z}[x]$.

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4. Let D be an integral domain and $Q(D)$ the field of fractions of D. Let $\varphi : D \to Q(D)$ be the map defined by $\varphi(d) = [d, 1]$.

   (a) Show that φ is a ring homomorphism.

   (b) Show that φ is injective.

   (c) Show that if D is a field, then φ is surjective (hence $D \cong Q(D)$ in this case).

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5. Let $\varphi : R \to D$ be a ring homomorphism, where R is a commutative ring and D is an integral domain. Show that the ideal $\ker\varphi$ of R is a *prime* ideal. [You need not show $\ker\varphi$ is an *ideal*.]