

MATH 4/51002
Homework #8
Due: Friday, April 6

Reading:

For Monday, April 2: §6.1
For Friday, April 6: §6.2

Problems to turn in:

In all problems, V is a finite dimensional vector space over a field F .

1. Let $\mathbf{v}_1, \dots, \mathbf{v}_n, \mathbf{w} \in V$. Show that

$$\text{sp}\{\mathbf{v}_1, \dots, \mathbf{v}_n, \mathbf{w}\} = \text{sp}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$$

if and only if \mathbf{w} is a linear combination of the vectors $\mathbf{v}_1, \dots, \mathbf{v}_n$.

2. Show that no linearly independent set of vectors in V can contain the zero vector.
3. Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly independent subset of V and let

$$T = \{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_1 + \mathbf{v}_3\}.$$

- (a) Show that if $\text{char } F \neq 2$, then T is linearly independent.
(b) Show that if $\text{char } F = 2$, then T is *not* linearly independent.
4. Show that if a subset S of V is linearly independent, then any nonempty subset T of S is also linearly independent.
5. Show that if a subset S of V is linearly independent and $\mathbf{v} \in V$ is not in $\text{sp}(S)$, then $S \cup \{\mathbf{v}\}$ is linearly independent.
6. Let $\varphi : V \rightarrow W$ be a linear transformation. Show that if φ is surjective and $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ spans V , then $\{\varphi(\mathbf{v}_1), \dots, \varphi(\mathbf{v}_n)\}$ spans W .
7. Let $\varphi : V \rightarrow W$ be a linear transformation. Show that if φ is injective and $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ is a linearly independent subset of V , then $\{\varphi(\mathbf{v}_1), \dots, \varphi(\mathbf{v}_n)\}$ is a linearly independent subset of W .
8. Let W be a finite dimensional subspace of V .
- (a) Show that $\dim W \leq \dim V$.
(b) Show that if $\dim W = \dim V$, then $W = V$.