Math 4/51002
Exam II Review

Exam II will be given in class on **Friday, March 16, 2007**.

It will cover Sections 5.1 – 5.4 of the text, Homeworks 4 – 6, and material from class February 5 through March 7.

Topics to be covered include:

- **Rings** – Definition of ring and commutative ring; examples, including $\mathbb{Z}_n$, polynomial rings, matrix rings, Hamilton's Quaternions, and Gaussian integers; division rings and fields; basic properties of rings; subrings, definition, criteria, and examples; units and zero divisors, properties, examples, group of units; integral domain, definition and examples, cancellation properties, finite integral domains; direct sum of rings; characteristic of a ring, characteristic of an integral domain or field.

- **Ring Homomorphisms** – Definition of ring homomorphism, basic properties; examples, including field isomorphisms, evaluation homomorphism, reduction modulo n in $\mathbb{Z}$ or $\mathbb{Z}[x]$; ring isomorphism, definition and basic properties, "isomorphic" as an equivalence relation; homomorphisms and units, homomorphisms between groups of units; image and kernel of a ring homomorphism, relation to surjectivity and injectivity, ideal property of kernels, kernel of evaluation homomorphism.

- **Ideals and Quotient Rings** – Definition of ideal, examples, principal ideals (in commutative rings), ideals and fields; PID, definition, examples, non-examples; quotient ring, definition and examples; isomorphism theorems, canonical homomorphism, First Isomorphism Theorem, Correspondence Theorem; maximal and prime ideals, definitions, examples, non-examples, characterizations in terms of quotients, maximal and prime ideals in $F[x]$ and $\mathbb{Z}$, conditions under which (0) is a prime ideal or maximal ideal, relation between prime and maximal ideals, prime and maximal ideals in a PID.

- **Field of Fractions of an Integral Domain** – Definition of the field of fractions $Q(D)$, operations, calculations, examples; minimality property; prime subfield of a field.

Topics NOT covered:

- Definition of polynomial rings as given in Example 5.1.2
- Extending ring homomorphisms on R to $R[x]$ (Proposition 5.2.7)
- Direct sums of more than two rings (Proposition 5.2.8, Definition 5.2.9)
- Precise version of minimality of field of fractions (Theorem 5.4.6)

Possible types of questions:

- Statements of definitions and/or theorems, taken from the following:

Ring	Integral Domain	Quotient Ring
Commutative Ring	Characteristic	First Isomorphism Theorem
Division Ring	Ring Homomorphism	Correspondence Theorem
Subring	Ideal	Maximal Ideal
Unit	Principal Ideal	Prime Ideal
Zero Divisor	Principal Ideal Domain (PID)	Field of Fractions

- Computational Problems:
  - Addition and multiplication in rings, including Hamilton's Quaternions $\mathbb{H}$
  - Finding units and inverses in rings, including $\mathbb{H}$
  - Finding zero divisors in rings
  - Computations involving ring homomorphisms
  - Computing kernels and images of ring homomorphisms
  - Characteristic of a ring
  - Addition, multiplication, units, inverses, zero divisors in quotient rings
  - Computing the field of fractions of an integral domain D
  - Addition, multiplication, and inverses in fields of fractions
- Proofs similar to those in homework, including:
  - General basic properties of rings
  - Verifying subsets are subrings
  - Basic properties of integral domains
  - Verifying sets of units or zero divisors of a given ring
  - Verifying maps are ring homomorphisms or isomorphisms
  - Properties of homomorphisms
  - Show two rings are or are not isomorphic
  - Properties of the characteristic of a ring
  - Verifying subsets are ideals
  - Verifying an ideal is or is not maximal or prime
  - General properties of ideals, maximal ideals, prime ideals
  - Properties of Principal Ideal Domains
  - Properties of quotient rings
  - Using First Isomorphism Theorem to prove ring isomorphisms
  - Properties of fields of fractions and their operations