(3) Let t be an indeterminate (variable) and denote

$$P(t) = \{a_0 + a_1 t + a_2 t^2 + \cdots + a_m t^m \mid m \geq 0, a_i \in \mathbb{R}\}.$$

$P(t)$ is the set of all polynomials in t with coefficients in $\mathbb{R}$. This is a vector space over $\mathbb{R}$ under the usual "termwise" polynomial addition and scalar (constant) multiplication:

$$(a_0 + a_1 t + \cdots + a_s t^s) + (b_0 + b_1 t + \cdots + b_s t^s)$$
$$= (a_0 + b_0) + (a_1 + b_1)t + \cdots + (a_s + b_s)t^s$$

and $\alpha(a_0 + a_1 t + \cdots + a_s t^s) = (\alpha a_0) + (\alpha a_1)t + \cdots + (\alpha a_s)t^s$.

Note that $P(t)$ consists of polynomials over $\mathbb{R}$ of all degrees (m is not fixed, but ranges over all nonnegative integers).

(4) A very important subset of $P(t)$ is obtained by fixing a nonnegative integer n and taking the set $P_n(t)$ of all polynomials in $P(t)$ of degree $\leq n$:

$$P_n(t) = \{a_0 + a_1 t + \cdots + a_n t^n \mid a_i \in \mathbb{R}\}.$$

[For example, $P_3(t) = \{a_0 + a_1 t + a_2 t^2 + a_3 t^3 \mid a_i \in \mathbb{R}\}$.]

$P_n(t)$ is also a vector space over $\mathbb{R}$ with the same operations as in $P(t)$.

It is important to observe that $P_n(t)$ is closed under both operations (WHY?).

[See text for other examples.]