Isomorphisms

Defn: A linear transformation $\varphi: V \to W$ is an <u>isomorphism</u> if φ is both one-to-one and onto. In this case, we say V is <u>isomorphic</u> to W and write $V \cong W$.

Isomorphic vector spaces are "algebraically" the same, except for notation of vectors.

EXAMPLE: If V is a vector space over K of dimension n and basis B, then $\varphi: V \to K^n$ given by $\varphi(\bar{v}) = [\bar{v}]_B$ is an isomorphism. Hence if $\dim V = n$ then $V \cong K^n$.

In particular, $P_n(t) \cong \mathbb{R}^{n+1}$ and $M_{m,n}(\mathbb{R}) \cong \mathbb{R}^{mn}$.

Recall that if $\varphi: V \to W$ is <u>any</u> map that is <u>one to one</u> and <u>onto</u>, there is an inverse map $\varphi^{-1}: W \to V$ defined by $\varphi^{-1}(\bar{w}) = \bar{v}$ if and only if $\varphi(\bar{v}) = \bar{w}$.
Moreover, if φ is <u>linear</u>, then φ^{-1} is also linear:

Proposition: If $\varphi: V \to W$ is a linear transformation that is one to one and onto, then the inverse map $\varphi^{-1}: W \to V$ is a linear transformation.

$\boxed{\text{End Lec 17}}$ **Pf:** [See Problem 5.15] $\square$

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Kernel and Image (§5.4)

Defn: Let $\varphi: V \to W$ be a linear transformation.
The <u>Kernel of φ</u> is $\ker \varphi = \{\bar{v} \in V \mid \varphi(\bar{v}) = \bar{0}_W\}$, the set of vectors <u>of V</u> mapped to $\bar{0}_W$ by φ.
The <u>image of φ</u> is $\operatorname{Im} \varphi = \{\bar{w} \in W \mid \bar{w} = \varphi(\bar{v}) \text{ some } \bar{v} \in V\}$, the set of vectors <u>of W</u> that are the image of some vector in V under φ.