EXAMPLES:

1. $$A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$$

$$\Delta_A(t) = \begin{vmatrix} t-1 & -2 & 2 \\ 2 & t-5 & 2 \\ 6 & -6 & t+3 \end{vmatrix} = t^3 - 3t^2 - 9t + 27 = (t-3)^2(t+3)$$

[Exercise: Check.]

Eigenspace for $\lambda = -3$:

$E_{-3} = \mathcal{N}(-3I - A)$ (Substitute $t = -3$ in $tI - A$),

$$-3I - A = \begin{bmatrix} -4 & -2 & 2 \\ 2 & -8 & 2 \\ 6 & -6 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 1 \\ -4 & -2 & 2 \\ 6 & -6 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 1 \\ 0 & -18 & 6 \\ 0 & 18 & -6 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence E_{-3} is the set of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying
$$x - 4y + z = 0$$
$$-3y + z = 0$$

Thus z is free, $y = \frac{1}{3}z$, $x = 4y - z = \frac{4}{3}z - z = \frac{1}{3}z$.

Hence $E_{-3} = \left\{ \begin{bmatrix} \frac{1}{3}z \\ \frac{1}{3}z \\ z \end{bmatrix} \middle| z \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} z \\ z \\ 3z \end{bmatrix} \middle| z \in \mathbb{R} \right\}$, and $\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}$ is a basis, $\underline{\dim E_{-3} = 1}$.

Eigenspace for $\lambda = 3$:

$E_3 = \mathcal{N}(3I - A)$

$$3I - A = \begin{bmatrix} 2 & -2 & 2 \\ 2 & -2 & 2 \\ 6 & -6 & 6 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Thus E_3 is the set of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying $x - y + z = 0$.

Hence y, z are free, $x = y - z$, and so

$E_3 = \left\{ \begin{bmatrix} y-z \\ y \\ z \end{bmatrix} \middle| y, z \in \mathbb{R} \right\}$ and $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis, $\underline{\dim E_3 = 2}$.

$\boxed{\text{END LEC 25}}$ [Exercise: Verify that $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ are eigenvectors belonging to $\lambda = 3$.]