(84) Lec 29, 4/10/09

(EXAMPLES, cont.)

(2) $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ we showed previously (4/3/09, page 75) That $\Delta_A(t) = (t-1)^2(t-2)$ and that $\dim E_1 = 1$.

Since $\lambda = 1$ is a root of $\Delta_A(t)$ of multiplicity 2 and $\dim E_1 = 1 < 2$, we know that A is not diagonalizable.

[Recall: $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$ is a basis for E_1, $\left\{\begin{bmatrix} -7 \\ 4 \\ 1 \end{bmatrix}\right\}$ is a basis for E_2.

Hence $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 4 \\ 1 \end{bmatrix}\right\}$ is the largest possible linearly independent set of eigenvectors of A.]

(3) $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$, so that $tI - A = \begin{bmatrix} t-1 & -2 & 2 \\ 2 & t-5 & 2 \\ 6 & -6 & t+3 \end{bmatrix}$.

We showed previously (3/30/09, page 74) that

$$\Delta_A(t) = (t-3)^2(t+3),$$

$E_3 = \eta\left(\begin{bmatrix} 2 & -2 & 2 \\ 2 & -2 & 2 \\ 6 & -6 & 6 \end{bmatrix}\right)$, basis for E_3 is $\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$,

$E_{-3} = \eta\left(\begin{bmatrix} -4 & -2 & 2 \\ 2 & -8 & 2 \\ 6 & -6 & 0 \end{bmatrix}\right)$, basis for E_{-3} is $\left\{\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}\right\}$.

Hence $\dim E_3 = 2$, $\dim E_{-3} = 1$, and in both cases the dimension is equal to the multiplicity of the eigenvalue as a root of $\Delta_A(t)$. Therefore, A is diagonalizable.

If $P = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$, so $P^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 4 & -1 \\ -3 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix}$, then $PAP^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$.