Math 4/52021
Exam I Review

Exam I will be given in class on **Thursday, October 8, 2009**. It will cover Chapters 1 and 2, Homework #1–#4, and material from class through September 29. The problems will be similar to homework problems.

Listed below are the basic terminology and results you should be familiar with and the types of problems and skills to be tested. Proofs of basic results related to the listed terminology and theorems may be included. Statements of definitions/theorems and proofs of theorems will *not* be on the exam.

Terminology

§1.1: vertex, edge, graph, adjacent, incident, directed graph, path, circuit, matching, independent set

§1.2: isomorphism (undirected graphs only), degree of a vertex, subgraph, complement, complete graph K_n

§1.3: bipartite graph

§1.4: planar graph, plane graph, dual graph, region, complete bipartite graph $K_{m,n}$

§2.1: cycle, trail, Euler cycle, Euler trail, multigraph, line graph $L(G)$

§2.2: Hamilton circuit, Hamilton path

§2.3: k-coloring, chromatic number $\chi(G)$, chromatic polynomial, edge coloring

Theorems

Chapter 1:

- Features isomorphic graphs necessarily share (including number of edges, number of vertices, number of vertices of each degree, subgraphs, circuits).
- Two graphs are isomorphic if and only if their complements are isomorphic.
- Sum of degrees is $2e$.
- The number of edges of odd degree is even.
- A graph is bipartite if and only if every circuit is of even length.
- Euler's formula.
- If G is connected and planar then $e \leqslant 3v - 6$.

Chapter 2:

- Necessary and sufficient conditions for existence of an Euler cycle or Euler trail.
- Basic rules only (page 57); *not* Theorems 1–4.
- Chromatic numbers of basic graphs (bipartite, complete, circuit, wheel).
- Relation between chromatic number and chromatic polynomial.
- 4-Color Theorem.

Types of Problems

Chapter 1:

- Modeling using graphs.
- Finding a matching or proving none exists in a given bipartite graph.
- Determine (and prove) whether two graphs are or are not isomorphic.
- Find the complement of a graph.
- Counting/parity problems (regarding vertices, edges), including proving non-existence of graphs with certain parameters.
- Determine whether a graph is or is not bipartite.
- Use circle-chord method to show that a graph is not planar.
- Find the dual of a graph G.
- Find e, v, or r given two of the three.
- Determine if a graph with given parameters can be planar.
- Find the line graph $L(G)$ of a graph G.

Chapter 2:

- Determine if an Euler cycle or trail exists and find one if it does exist.
- Determine whether a given graph has a Hamilton circuit. Give a proof if not, and find one if one exists.
- Find the chromatic number and/or chromatic polynomial for a given graph.
- Proofs of basic results; for example, similar to those in §2.4 problems.