MATH 6/71051
COSETS

Let G be a group, H a subgroup of G, and g an element of G.

Definition 1. *The* **left coset** *of H in G containing g is the set $gH = \{gh \mid h \in H\}$.*

Definition 1′. *The* **right coset** *of H in G containing g is the set $Hg = \{hg \mid h \in H\}$.*

LEFT COSETS

We proved the following results for *left* cosets in class.

Theorem 1. *If $H \leqslant G$ and $g \in G$, then $gH = H$ if and only if $g \in H$.*

Theorem 2. *If $H \leqslant G$ and $a, b \in G$, then either $aH \cap bH = \emptyset$ or $aH = bH$.*

Thus the left cosets of H in G partition G.

Theorem 3. *Let $H \leqslant G$ and let $a, b \in G$. The following are equivalent:*

i. $aH \cap bH \neq \emptyset$

ii. $aH = bH$

iii. $b \in aH$

iv. $a \in bH$

v. $b = ah$ *for some* $h \in H$

vi. $a = bk$ *for some* $k \in H$

vii. $a^{-1}b \in H$

viii. $b^{-1}a \in H$.

RIGHT COSETS

Prove the following analogous results for *right* cosets as an exercise.

Theorem 1′. *If $H \leqslant G$ and $g \in G$, then $Hg = H$ if and only if $g \in H$.*

Theorem 2′. *If $H \leqslant G$ and $a, b \in G$, then either $Ha \cap Hb = \emptyset$ or $Ha = Hb$.*

Thus the right cosets of H in G partition G.

Theorem 3′. *Let $H \leqslant G$ and let $a, b \in G$. The following are equivalent:*

i. $Ha \cap Hb \neq \emptyset$

ii. $Ha = Hb$

iii. $b \in Ha$

iv. $a \in Hb$

v. $b = ha$ *for some* $h \in H$

vi. $a = kb$ *for some* $k \in H$

vii. $ba^{-1} \in H$

viii. $ab^{-1} \in H$.