

MATH 6/71051  
Homework #10  
**Due: Thursday, November 15**

**Reading:**

For Thurs., November 15: §3.4  
§6.2 & §6.3 [will not be covered in class]

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**Problems to turn in:**

I. (a) Let  $H$  and  $K$  be subgroups of  $G$  and  $\varphi : G \rightarrow G_1$  a homomorphism. Show that

$$\varphi([H, K]) = [\varphi(H), \varphi(K)].$$

Use induction to deduce that  $\varphi(G^i) = \varphi(G)^i$  for all  $i \geq 0$ .

(b) Let  $N \trianglelefteq G$  and  $N \leq H \leq G$ . Show that  $H/N \leq Z(G/N)$  if and only if  $[H, G] \leq N$ .

**§6.1:** 7, 14, 25, 26(a).

II. Let  $G$  be a finite group. Show that if  $N \trianglelefteq G$ ,  $N \leq \Phi(G)$ , and  $G/N$  is cyclic, then  $G$  is cyclic. In particular, deduce that if  $G/\Phi(G)$  is cyclic, then  $G$  is cyclic.

III. Let  $H$ ,  $J$ , and  $K$  be normal subgroups of  $G$  with  $J \leq H$ . Show that if  $H/J \leq Z(G/J)$ , then  $HK/JK \leq Z(G/JK)$ .

IV. Show that the lower central series of a group  $G$  is a central series.

V. Show that if  $G$  is a nonabelian group of order  $p^3$ ,  $p$  a prime, then  $\Phi(G) = G' = Z(G)$ .

VI. Show that if  $G$  is a finite nilpotent group and  $G/G'$  is cyclic, then  $G$  is cyclic.

**Notes:**

- Hint for §6.1 #7 and #14: Use Problem #I(a).
  - Hint for Problem #III: Use Problem #I(b).
  - On Problem #IV: Regardless of how credible you may consider your instructor to be, it does NOT follow that the lower central series is a central series just because he called it the lower *central series*.
  - On Problem #V: Recall that we have shown  $G' = Z(G)$  already (Homework #9, Problem #II), so it is only necessary to show  $\Phi(G) = G'$  or  $\Phi(G) = Z(G)$ .
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**Problems to be aware of:**

§6.1: **1, 3, 4, 6, 12, 13, 21, 22, 24.**

[Problems in **bold** contain results we will assume and use in the course.]