Reading:

For Monday, October 2: §4.1
For Wednesday, October 4: §4.2
For Friday, October 6: §4.3

Problems to turn in:

§3.1: 36, 40, 41.
§3.2: 4.
§3.3: 7.

I. Let $G$ be a group, Aut($G$) the automorphism group of $G$, and Inn($G$) the group of inner automorphisms of $G$.
   (a) Show that Inn($G$) $\triangleleft$ Aut($G$).
   (b) Show that $G/Z(G)$ $\cong$ Inn($G$).

II. Let $H \leq G$.
   (a) Show that $C_G(H) \triangleleft N_G(H)$.
   (b) Show that for $g \in N_G(H)$, the map $\sigma_g : H \to H$ given by $\sigma_g(h) = ghg^{-1}$ is an automorphism of $H$.
   (c) Show that $N_G(H)/C_G(H)$ is isomorphic to a subgroup of Aut($H$).

III. Let $\varphi : G \to H$ be a group homomorphism and let $K = \ker \varphi$.
   (a) Show that if $M \leq G$, then $\varphi^{-1}(\varphi(M)) = MK$.
   (b) Show that if $\varphi$ is surjective and $L \leq H$, then $\varphi(\varphi^{-1}(L)) = L$.

IV. Let $\varphi : G \to H$ be a surjective homomorphism. Show that if $N \unlhd G$, then $\varphi(N) \unlhd H$.

V. Let $\varphi : G \to H$ be a homomorphism and let $\ker \varphi \leq K \leq G$. Show that if $\varphi(K) \leq H$, then $K \unlhd G$.

Notes: See second page.
Notes:

- On Problems #I(b) and #II(c), use the First Isomorphism Theorem.

- On Problem #II(a), note that elements of $C_G(H)$ commute with elements of $H$ but not necessarily with elements of $N_G(H)$.

- On Problem #II(b), the automorphism $\sigma_g$ defined here is not necessarily an inner automorphism, since $g$ need not be an element of $H$. The element $g$ must be in $N_G(H)$ in order for $\sigma_g$ to map $H$ to $H$.

- On Problem #V, you may want to use #III(a).

- Problems #III–V are used in the proof of the Correspondence Theorem. Do not use the Correspondence Theorem to prove any of these statements.

Problems to be aware of:

§3.1: 3, 4, 5, 16, 32, 33, 37, 38.
§3.2: 5.
§3.3: 1, 3, 4.
§3.5: 2, 3, 4, 12.

[Problems in bold contain results we will assume and use in the course.]