# Algebra and Representation HW #3

For each algorithm described below, determine whether the method works in all cases. If so, explain how you know the method works and explain *why* it works. If not, give an example where the method fails.

1. **Divide two-digit numbers.**
   When the ones digit of one equals the tens digit of the other, divide by simply "cancelling" the equal digits. Examples:

$$\frac{6\!\!\!/4}{1\!\!\!/0} = \frac{4}{1} = 4 \qquad \frac{19\!\!\!/}{9\!\!\!/5} = \frac{1}{5} \qquad \frac{6\!\!\!/5}{2\!\!\!/0} = \frac{5}{2} \qquad \frac{9\!\!\!/8}{49\!\!\!/} = \frac{8}{4} = 2.$$

2. **Square a two-digit number ending in $5$.**

   Example: $65^2$.

   Step 1: Multiply the tens digit by the number one more than itself: $6 \times 7 = 42$.

   Step 2: Append 25 on the end of the result of Step 1: $65^2 = 4225$.

3. **Square any two-digit number.**
   (Generalizes the previous method.)

   Example: $83^2$.

   Step 1: Multiply the tens digit by itself and append two zeros: $8 \times 8 = 64 \longrightarrow 6400$.

   Step 2: Multiply the ones digit by itself and add to the result of Step 1:

   $$3 \times 3 = 9 \longrightarrow 6400 + 9 = 6409.$$

   Step 3: Multiply the tens digit by twice the ones digit and append a zero:

   $$8 \times (2 \times 3) = 48 \longrightarrow 480.$$

   Step 4: Add the results of Step 2 and Step 3: $6409 + 480 = 6889 = 83^2$.