

## Algebra and Representation HW #7

### I. Verifying Magic Square Properties

1. Suppose we have a 3 by 3 Magic Square, called Square 1, and that the three numbers in some row, column, or diagonal are represented by the variables  $a$ ,  $b$ , and  $c$ . Also, represent the magic number by the variable  $M$ .

Now suppose we *multiply* each entry of Square 1 by the same number, represented by the variable  $p$ . Call this new square Square 2.

- (a) In terms of the variables above, what are the entries in the row, column, or diagonal of Square 2 corresponding to those of Square 1?
  - (b) Write an expression for the *sum* of these entries of Square 2 in terms of the variables  $a$ ,  $b$ , and  $c$ .
  - (c) Now write this sum in terms of the magic number  $M$  of Square 1.
  - (d) Use this to explain why Square 2 must be a Magic Square. What is the magic number of Square 2?
2. We now consider the difference of two Magic Squares. Again, we let  $a$ ,  $b$ , and  $c$  represent the entries in a row, column, or diagonal of Square 1 and let  $M$  represent the magic number of Square 1. Represent the *corresponding* entries of Square 2 with the variables  $x$ ,  $y$ , and  $z$ , respectively, and represent the magic number of Square 2 by  $N$ .

Now *subtract* Square 2 from Square 1 by subtracting corresponding entries to obtain a new square, Square 3.

- (a) In terms of the variables above, what are the entries in the row, column, or diagonal of Square 3 corresponding to those of Square 1 and Square 2?
- (b) Write an expression for the *sum* of these entries of Square 3 in terms of the variables  $a$ ,  $b$ ,  $c$ ,  $x$ ,  $y$ , and  $z$ .
- (c) Now write this sum in terms of the magic numbers  $M$  and  $N$ .
- (d) Use this to explain why Square 3 must be a Magic Square. What is the magic number of Square 3?

### II. Formulas and Equations

3. The volume of a rectangular prism is given by  $V = \ell \times w \times h$ , where  $\ell$  is the length,  $w$  is the width, and  $h$  is the height of the prism.
  - (a) Find the volume if the length is 3 feet, width is 7 feet, and the height is 4.5 feet.
  - (b) Find a formula for the height  $h$  of a rectangular prism in terms of the volume  $V$ , length  $\ell$ , and width  $w$ . Use your formula to find the height of a prism with volume  $52 \text{ cm}^3$ , length 4 cm, and width 3 cm.
4. The volume of a (circular) cylinder is given by  $V = \pi r^2 h$ , where  $r$  is the radius of the base and  $h$  is the height of the cylinder. Use this to find a formula for the radius in terms of the volume and the height.