FUNCTIONS, CONTINUED: SYMBOLIC REPRESENTATIONS

VIII. Symbolic Representations of Functions

Functions are often represented symbolically. The dependent variable is given in terms of some algebraic expression involving the independent variable. This is similar to what we did when we found an **explicit** formula for the n^{th} term of a number pattern in terms of *n*. We view the pattern as a function, f , with domain the set of counting numbers and with $f(n)$ denoting the n^{th} term in the pattern.

Examples:

A. The function f giving the number of external sides in a "square train" with *n* squares is given by $f(n) = 2n + 2$.
B. The function f giving the number of external sides in a "hexagon train" with *n* hexagons is given by $f(n) = 4n + 2$.
C. The function f assigning to each counting number *n* the sum of the first *n* **odd** counting numbers is given by $f(n) = n^2$.
D. The function f assigning to each counting number *n* the sum of the first *n* counting numbers is given by $f(n) = \dfrac{n(n+1)}{2}$.
E. The function *C* giving the circumference of a circle of radius *r* is given by $C(r) = 2\pi \cdot r$.
F. The function *A* giving the area of a circle of radius *r* is given by $A(r) = \pi \cdot r^2$.

Exercises:

1. For each function above:
   a. Make a **table** of function values for *n* or *r* from 1 to 10 (counting numbers).
   b. Carefully sketch a **graph** of the function, accurately plotting the points from the table, and then "connect the dots" with a smooth curve. (For A – D, this extends the domain to all **real** numbers between 1 and 10.)

2. Now compute the difference between each consecutive pair of function values from the tables in 1(a).
   a. Compare the graphs and differences in consecutive function values for the functions in A, B, and E above to those for the functions in C, D, and F. What do you notice?
   b. Make a conjecture about what distinguishes a straight line graph from other types of graphs.

What are some of the advantages and some of the disadvantages of giving an algebraic representation of a function? In what situations might it be preferable to other types of representations?

FUNCTIONS: SYMBOLIC REPRESENTATIONS, CONTINUED

IX. Function Notation and Recursive Formulas

Recall that a **recursive** formula for a pattern is a rule that allows us to find the value of one term in the pattern if we know the previous term. For example, in the pattern

$$\{1, 3, 7, 15, 31, 63 \ldots\},$$

notice that each term is one more than twice the previous term. A possible recursive formula for this pattern is

$$(n + 1)^{\text{th}} \text{ term} = 2 \text{ times } n^{\text{th}} \text{ term} + 1.$$

The use of functional notation provides a more concise, less awkward, and less ambiguous method for writing recursive formulas. As noted above, we may view patterns as functions whose domains are the set of counting numbers, so that the n^{th} term in the pattern is $f(n)$. In the example above, we would have:

n	1	2	3	4	5	6	…
$f(n)$	1	3	7	15	31	63	…

Using this notation, the recursive formula above would become

$$f(n+1) = 2f(n) + 1.$$

Recall that an **explicit** formula for a pattern is a formula that gives the n^{th} term in the pattern in terms of n. An explicit formula for the pattern in this example is

$$f(n) = 2^n - 1.$$

Exercises:

Use functional notation to write a **recursive** formula for each of the following patterns.

1. The pattern whose n^{th} term is the number of "external sides" for a hexagon train with n hexagons:

n	1	2	3	4	5	6	…
$f(n)$	6	10	14	18	22	26	…

2. The pattern whose n^{th} term is the number of "sheets" obtained by folding a piece of paper n times:

n	1	2	3	4	5	6	…
$f(n)$	2	4	8	16	32	64	…

3. The pattern whose n^{th} term is the sum of the first n counting numbers:

n	1	2	3	4	5	6	…
$f(n)$	1	3	6	10	15	21	…

4. The pattern whose n^{th} term is the sum of the first n **odd** counting numbers:

n	1	2	3	4	5	6	…
$f(n)$	1	4	9	16	25	36	…