Properties of Operations of Arithmetic

I. Why Do the Algorithms Work?

Traditional and nontraditional algorithms for some operations from arithmetic are demonstrated below. For each algorithm, use the basic properties of addition and multiplication (such as commutative, associative, and distributive laws) to **explain** why the algorithm works. Also, find another method for doing the computation and explain why your method works.

1. **Addition**

   (a) Addition with "carrying":

$$\begin{array}{r} {\scriptstyle 1} \\ 46 \\ +\,38 \\ \hline 84 \end{array}$$

   (b) Left to right addition:

$$\begin{array}{r} 46 \\ +\,38 \\ \hline 7 \\ +\,14 \\ \hline 84 \end{array}$$

   Which method do you prefer?
   Which method would seem more "natural" if you were learning to add 2-digit numbers for the first time?

2. **Subtraction**

   (a) Subtraction with "borrowing":

$$\begin{array}{r} {\scriptstyle 4\ 17} \\ \not{5}\,\not{7} \\ -3\ 9 \\ \hline 1\ 8 \end{array}$$

   (b) Subtraction by "adding on":

$$57 - 39 \longrightarrow 58 - 40 = 18$$

(**I. Why Do the Algorithms Work?**, continued)

3. **Multiplication**

   (a) Standard algorithm:

$$\begin{array}{r} 231 \\ \times\ \ 32 \\ \hline 462 \\ 6\,93 \\ \hline 7\,392 \end{array}$$

   (In particular, WHY is 693 shifted?)

   (b) Left to right multiplication:

$$\begin{array}{r} 357 \\ \times\ \ \ 8 \\ \hline 2\,400 \\ 400 \\ 56 \\ \hline 2\,856 \end{array}$$

   (In particular, where do the zeros come from?)

   (c) Multiplication using compensation:

   Multiply any two digit number by 99:

$$99 \times 36 = 3600 - 36 = 3564$$

$$99 \times 73 = 7300 - 73 = 7227$$

   This also works with other products:

$$37 \times 13 = (40 \times 13) - (3 \times 13) = 520 - 39 = 481$$

II. Unconventional Algorithms

Shortcuts or nonstandard methods for doing some calculations are described below. For each algorithm, determine whether the method works in all cases. If so, explain how you know the method works and explain *why* it works. If not, give an example where the method fails.

1. **Multiplying Two-digit Numbers up to** 19×19

   Example: 17×14.

   Step 1: Add one of the numbers to the ones digit of the other and multiply by 10:

   $$17 + 4 = 21 \to 210.$$

   Step 2: Multiply the ones digits: $7 \times 4 = 28$.

   Step 3: Add the results of Step 1 and Step 2: $210 + 28 = 238 = 17 \times 14$.

   Does this method work for 2-digit numbers larger than 20? Why or why not?

2. **Multiplying a Mixed Number by a Whole Number**

   Example: $9\frac{3}{4} \times 8$.

   Step 1: Multiply the whole numbers: $9 \times 8 = 72$.

   Step 2: Multiply the fraction by the whole number: $\frac{3}{4} \times 8 = 6$.

   Step 3: Add the results of the first two steps: $72 + 6 = 78$.

   It may be necessary to write a fraction as a mixed number in Step 2.

   Example: $7\frac{5}{8} \times 9$.

   Step 1: Multiply the whole numbers: $7 \times 9 = 63$.

   Step 2: Multiply the fraction by the whole number: $\frac{5}{8} \times 9 = \frac{45}{8} = 5\frac{5}{8}$.

   Step 3: Add the results of the first two steps: $63 + 5\frac{5}{8} = 68\frac{5}{8}$.

3. **Division of Fractions**

   Step 1: Write both fractions with a common denominator.

   Step 2: Divide numerators.

   Example: $\dfrac{2}{3} \div \dfrac{5}{4} = \dfrac{8}{12} \div \dfrac{15}{12} = \dfrac{8}{15}$.