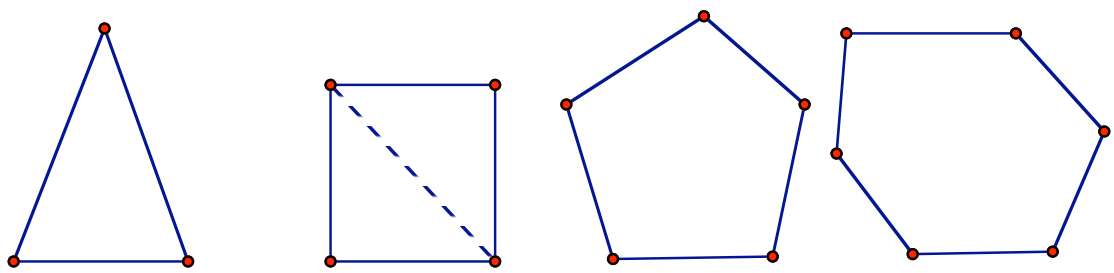
PATTERNS, CONTINUED: EXPLICIT FORMULAS

An **explicit formula** is a formula that enables you to compute any given term in a pattern without knowing the previous term; that is, a formula giving the nth term in terms of n.
In the example of {4, 11, 18, 25, 32…}, we might use the formula $4 + 7(n - 1)$ to find the nth term.

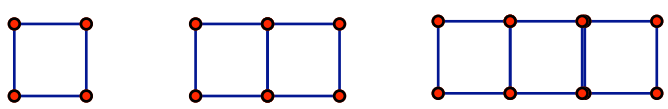
1. Recall the sequence of polygons in which the number of sides increases by one in each successive polygon. We divided the polygons into triangles by choosing a single vertex and drawing all possible diagonals from that vertex, and we considered the pattern of the number of triangles obtained.

   Determine an explicit formula for the number of triangles formed in a polygon with n sides. **Explain** how you got your formula.



2. Recall finding the number of "external sides" in the square trains previously.

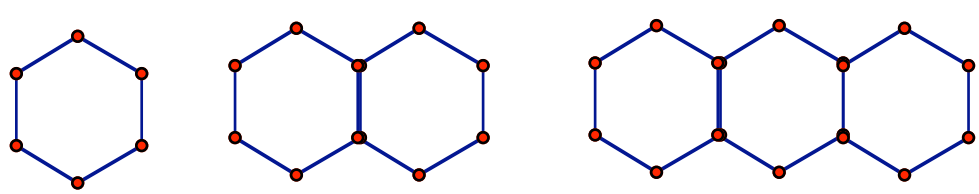
   Determine an explicit formula for the number of "external sides" in a train made up of n squares. **Explain** how you got your formula.



1st 2nd 3rd

3. Recall also finding the number of "external sides" in the hexagon trains.

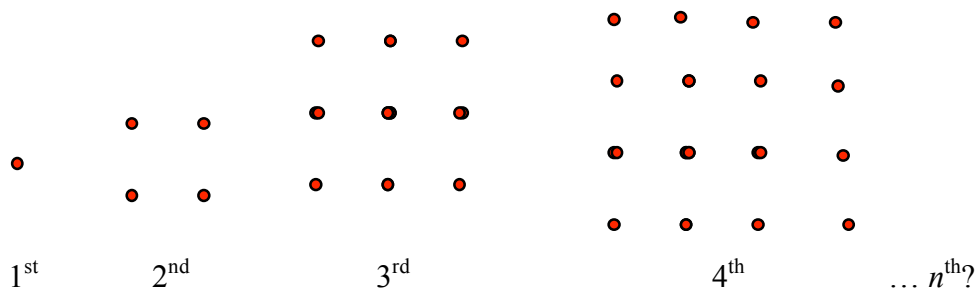
   Determine an explicit formula for the number of "external sides" in a train made up of n hexagons. **Explain** how you got your formula.



PATTERNS AND EXPLICIT FORMULAS, continued

4. **Old Patterns, Revisited**

   a. Find an explicit formula for the number of dots in the nth object below. **Explain** how you got your formula.



   b. Recall the "up and down sequences" we considered before, where the nth sequence consists of the consecutive integers starting at 1, going up to n, then back down to 1. Find an explicit formula for the *sum* of the nth sequence; for example, for $n = 5$ the sum of the sequence is $1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25$. Use the pattern of dots above to explain **why** your formula works. (Hint: Think diagonally.)

   c. Find an explicit formula to determine the sum of the first n odd counting numbers; for example, for $n = 3$, the sum is $1 + 3 + 5 = 9$. Compute the sum of the first 1, 2, 3, 4, and 5 odd counting numbers to help you get started. Use the pattern of dots above to explain **why** your formula works. (Hint: Think recursively.)

   d. Find an explicit formula for the sum of the first n counting numbers; for example, for $n = 4$, the sum is $1 + 2 + 3 + 4 = 10$. Compute the sum of the first 1, 2, 3, 4, and 5 counting numbers to help you get started. Use your formula to find the sum of the first 20 counting numbers.

      (Note: There are many ways to find this formula. You may be able to derive it from one of the formulas in (b) or (c), or use your own method to find the formula.)