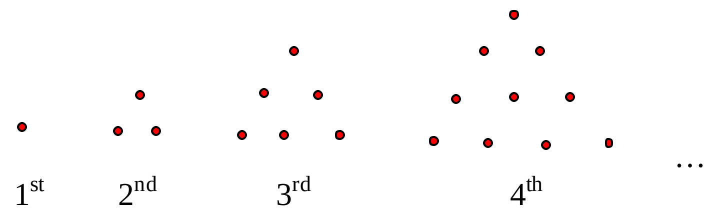
## PATTERNS AND VERIFYING FORMULAS, continued

2. Recall the **Triangular Numbers** problem: find the number of dots in the $n^{\text{th}}$ object below:




Step 1: We conjectured that the number of dots in the $n^{\text{th}}$ object, which is also the sum of the first $n$ counting numbers, is $n(n+1)/2$.

The following steps establish this formula for all counting numbers $n$:

Step 2: Verify that the formula works when $n = 1$:

a. How many dots are there in the first object?
b. What does the formula (that is, $n(n+1)/2$ ) give for the number of dots when $n = 1$?
c. Are your answers to (a) and (b) the same? If so, then the formula works when $n = 1$.

Step 3: Now determine the number of dots in the $(k+1)^{\text{th}}$ object, *assuming* the formula works for the $k^{\text{th}}$ object:

a. How many dots does the *formula* say are in the $k^{\text{th}}$ object? (Substitute $k$ for $n$ in the formula.)
b. Assuming the expression in Step 3(a), determine how many dots are in the $(k+1)^{\text{th}}$ object:
    i. How many dots are *added* in going from the $k^{\text{th}}$ object to the $(k+1)^{\text{th}}$ object?
    ii. Assuming the results of Step 3(a) and 3(b)(i), what is the total number of dots in the $(k+1)^{\text{th}}$ object?

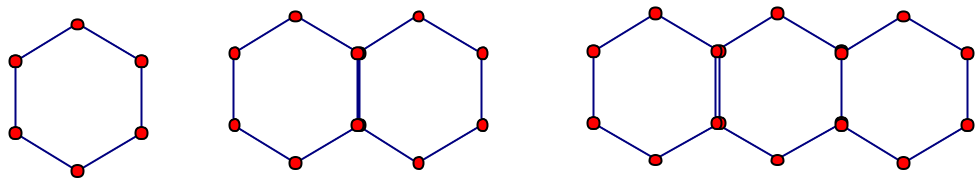
Step 4: How many dots does the *formula* say the $(k+1)^{\text{th}}$ object should have? (Substitute $k+1$ for $n$ in the formula.)

Step 5: Are your answers for the number of dots in the $(k+1)^{\text{th}}$ object from Step 3 and Step 4 the same? (Some algebraic manipulation may be needed in order to compare the two answers.)

The Principle of Mathematical Induction says that if the answers to Step 1(c) and Step 5 are both *yes*, then the formula must work for every counting number. That is, the number of dots in the $n^{\text{th}}$ object, or the sum of the first $n$ counting numbers, is $n(n+1)/2$.

### More Induction Problems

3. Recall the **Hexagon Trains** problem: find the number of "external sides" in the hexagon train made up of $n$ hexagons, as in the picture.

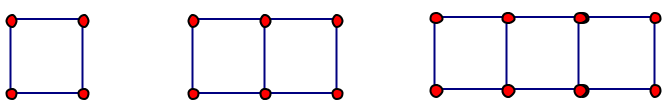


1st 2nd 3rd …

Use Mathematical Induction to show that the number of external sides in a hexagon train of $n$ hexagons is $4n+2$.

4. Use Mathematical Induction to show that the number of *all* sides in hexagon train made up of $n$ hexagons, as in the picture above, is $5n+1$.

5. Use Mathematical Induction to show that the number of *all* sides in square train made up of $n$ squares, as in the picture below, is $3n+1$.



1st 2nd 3rd …

6. Use Mathematical Induction to show that the $n^{\text{th}}$ *even* counting number is $2n$.

7. Use Mathematical Induction to show that the $n^{\text{th}}$ *odd* counting number is $2n-1$.