Algebra and Representation – Sample Exam Problems

Magic Squares

1. Determine whether each of the following is a magic square. If so, give the magic number. If not, explain why not.

9	4	5
2	6	10
7	8	3

16	2	12
5	10	15
9	18	3

1.5	4	0.5
1	2	3
3.5	0	2.5

2. Suppose you are given a 3 by 3 magic square M with magic number 15.
   a. If you add 4 to each entry of M, is the new square necessarily a magic square? If so, what is the magic number of the new square? If not, explain why not.
   b. If you multiply each entry of M by 7, is the new square necessarily a magic square? If so, what is the magic number of the new square? If not, explain why not.

Number Properties

3. Which property of number operations is demonstrated by each of the following equations?
   a. $3 + (2 + 5) = (3 + 2) + 5$.
   b. $3(2+5) = 3 \cdot 2 + 3 \cdot 5$.
   c. $7 \cdot 12 = 12 \cdot 7$.
4. State the commutative law of addition.
5. State the associative law of multiplication.

Guess the Number

6. Determine the relationship between the original number and the final number for the following procedure, and then **justify** your conclusion by using a variable in place of the chosen number.

   Step 1: Pick a number.
   Step 2: Add 3 to the number.
   Step 3: Multiply the result by 6.
   Step 4: Subtract 3.
   Step 5: Divide the result by 3.
   Step 6: Subtract 5 to obtain the final number.

Variables and Formulas

7. Recall that 1 yard is 3 feet. If F represents the number of feet and Y represents the number of yards, which of the following equations correctly relates F and Y?
   a. $Y = 3F$.
   b. $3Y = F$.

8. Recall that 1 mile is 5280 feet. If x represents the number of miles and y represents the number of feet, which of the following equations correctly relates x and y?
   a. $x = 5280y$.
   b. $5280x = y$.
   c. $y = \pi x^2$.

9. The formula for converting Celsius temperatures to Fahrenheit is

$$F = \frac{9}{5}C + 32,$$

   where C represents the number of degrees Celsius and F represents the number of degrees Fahrenheit.
   a. Find the Fahrenheit temperature if the temperature is 35 degrees Celsius.
   b. Find the Celsius temperature if the temperature is 18 degrees Fahrenheit.
   c. Use the formula above to find the formula for converting Fahrenheit temperatures to Celsius. Show your work.

Patterns, Recursive Formulas, and Explicit Formulas

10. Extend the pattern {3, 6, 12, 24, 48 …} for three more terms. Describe how you found these terms. (That is, give a *recursive* formula for the pattern.)

11. A number pattern starts with 5 and is described by the recursive formula
    $(n + 1)^{th}$ term = 3 times the n^{th} term.
    Find the next three terms in the pattern after 5 (i.e., the second, third, and fourth terms).

12. A number pattern is described by the recursive formula:
    $(n + 1)^{th}$ term = 1 more than 2 times the n^{th} term.
    If the 9^{th} term is 1279, find the 10^{th} and 11^{th} terms.

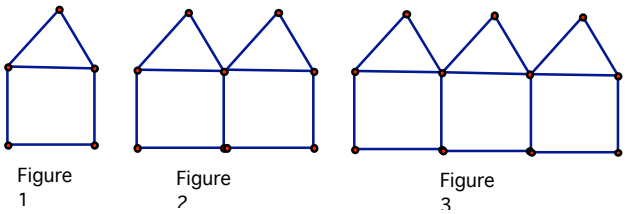
13. For the number pattern {1, 3, 5, 7, 9, 11…}, find an explicit formula that gives the n^{th} term as a function of n. Verify that your formula works for the first 6 numbers in the pattern. [NOTE: There are many, many possible correct answers. You only need to find a formula that works for the part of the pattern that is given.]

14. One way to describe how to get a number in Column B from the corresponding number in Column A is to "multiply by 2 and then add 8."

A	B
2	12
3	14
5	18
8	24

    What is another rule that could describe the same relationship?

15. In the pattern of "pentagon trains" below, each figure is obtained by adjoining one pentagon to the previous train.



   a. Find a recursive formula for the pattern of the **total** number of sides (internal and external) in a pentagon train; that is, find a formula for the $(n + 1)^{th}$ term in the pattern in terms of the n^{th} term.

   b. Find an explicit formula for the **total** number of sides in the n^{th} pentagon train. Use your formula to determine the total number of sides in the 27^{th} pentagon train.

16. Given a possible explicit formula for a number pattern, how many terms must be checked to verify that the formula is correct?

Functions

17. State a precise formal definition of *function.*

18. List four ways in which functions can be represented. For each type of representation give one advantage and one disadvantage of using that type of representation.

19. Give an example of a function for which the context would be something that is familiar to your students.

20. Give an example of a relation that is *not* a function, in a context that would be familiar to your students.

21. The length of a given rectangle is 4 inches. Give a table representation for the function A that assigns to each of the counting numbers w from 1 to 10 the area of the rectangle if the width of the rectangle is w inches.

w	1	2	3	4	5	6	7	8	9	10
$A(w)$										

Graphs and Formulas

22. Suppose you know that the point (3, 5) is on the graph of the function f. Determine the values of $f(3)$, $f(4)$, and $f(5)$, if possible. If it is *not* possible to determine some value, explain why not.

23. Explain why the point $(3, 9\pi)$ is on the graph of the function defined by the formula $A(r) = \pi \cdot r^2$. Find another point on the graph. (Assume the domain of the function is the set of all positive real numbers.)

24. Use graph paper to sketch the graph of the function f defined as follows.
    - f sends each real number between 0 and 3, inclusive, to the number 1.
    - f sends each real number between 4 and 8, inclusive, to the number 5.
    - f sends each real number between 9 and 10, inclusive, to the number 7.

    a. What are the domain and range of the function f?

    b. Without using a calculator, find the values of $f(\pi + 1)$, $f(3\pi)$, and $f(\sqrt{5})$.

25. Without graphing any points, determine which of the following tables, if any, represent points from a *linear* function. **Explain** your answers.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	5	6	8	11	15	20	26	33	41	50

x	1	2	3	4	5	6	7	8	9	10
$g(x)$	3	10	17	24	31	38	45	52	59	66

x	1	2	3	4	5	6	7	8	9	10
$h(x)$	2	3.5	5	6.5	7	8.5	10	11.5	13	14.5

x	1	2	3	4	5	6	7	8	9	10
$k(x)$	57	54	51	48	45	42	39	36	33	30

General

26. Explain how you might use "algebraic thinking" to aid in your teaching, even if you do not teach algebra.