Variables and Formulas, continued

III. Verifying Magic Square Properties

We previously investigated properties of Magic Squares by considering specific examples with numbers. We'll now use variables to further investigate these properties.

1. Suppose we have a 3 by 3 Magic Square, called Square 1, and that the three numbers in some row, column, or diagonal are represented by the variables a, b, and c. Suppose the magic number is represented by the variable M.

   (a) Write an equation relating these four variables.

   (b) Suppose we add 4 to each entry in Square 1 to obtain a new square. Call this Square 2.

      i. In terms of the variables above, what are the entries in the row, column, or diagonal of Square 2 corresponding to those of Square 1?

      ii. Write an expression for the *sum* of these entries of Square 2 in terms of the variables a, b, and c.

      iii. Now write this sum in terms of the magic number M of Square 1, using your equation from part (a).

      iv. Can you use this to explain why Square 2 must be a Magic Square? What is the magic number of Square 2?

   (c) Next, suppose we add the same number to each entry in Square 1 to obtain a new square. Represent this number by a variable n, and call the new square Square 3. Repeat (i) – (iv) above for Square 3.

2. We now consider the sum of two 3 by 3 Magic Squares. Again, we let a, b, and c represent the entries in a row, column, or diagonal of Square 1 and let M represent the magic number of Square 1. Represent the *corresponding* entries of Square 2 with the variables x, y, and z, respectively, and represent the magic number of Square 2 by N.

   (a) Write an equation relating the variables a, b, c, and M and another equation relating the variables x, y, z, and N.

   (b) Add Square 1 and Square 2 by adding corresponding entries, as we did before, to obtain a new square. Call this Square 3.

      i. In terms of the variables above, what are the entries in the row, column, or diagonal of Square 3 corresponding to those of Square 1 and Square 2?

      ii. Write an expression for the *sum* of these entries of Square 3 in terms of the variables a, b, c, x, y, and z.

      iii. Now write this sum in terms of the magic numbers M and N, using your equations from part (a).

      iv. Can you use this to explain why Square 3 must be a Magic Square? What is the magic number of Square 3?

Variables and Formulas, continued

IV. Formulas and Equations

A formula is an equation relating two or more variables that (usually) stand for some physical quantities. Some example of formulas you may have used before:

- $\boldsymbol{A = \ell \times w}$,
  where ℓ is the length of a rectangle, w is its width, and A is its area.
- $\boldsymbol{F = \frac{9}{5}C + 32}$,
  where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.
- $\boldsymbol{A = \frac{1}{2}bh}$,
  where b is the length of the base of a triangle, h is the height of the triangle, and A is the area of the triangle.
- $\boldsymbol{d = rt}$,
  where r is the (constant) rate or speed, t is the amount of time, and d is the distance travelled.
- $\boldsymbol{A = \pi r^2}$,
  where A is the area of a circle of radius r.
- $\boldsymbol{d = 2r}$
  where r is the radius of a circle and d is the diameter.

A. Substitution in Formulas

Given the values of the variables on the right hand side of one of these formulas, the values can be substituted to find the value of the variable on the left. Use the formulas above to solve the following problems. Be sure to use correct units in your answers.

1. Find the area of a rectangle with length 3.5 inches and width 4 inches.
2. The current temperature in my office is approximately $10°C$. Find the temperature in degrees Fahrenheit.
3. Find the area of a triangle with base of length 5 cm and height 7 cm.
4. If time is given in seconds and rate in meters per second, what are the appropriate units for distance?
5. Find the area of a circle of radius 2.5 feet. (Give an *exact* answer in terms of π as well as a decimal approximation.)