Verifying Magic Square Properties
Sample Proof

**Theorem.** Adding the same number \( n \) to each entry in a 3 by 3 magic square with magic number \( M \) yields a magic square with magic number \( M + 3n \).

**Proof.** Suppose we are given a 3 by 3 magic square, called Square 1, and the three numbers in some row, column, or diagonal are represented by the variables \( a, b, \) and \( c \).

\[
\begin{array}{c c c}
\text{Square 1} \\
| a | & | b | \\
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>&amp;</td>
</tr>
</tbody>
</table>
\end{array}
\]

Suppose further that the magic number for Square 1 is represented by the variable \( M \). It follows from the definition of a magic square that

\[
a + b + c = M
\]

and that this holds for any row, column, or diagonal with entries \( a, b, \) and \( c \).

Now suppose we add the same number \( n \) to each entry in Square 1 to obtain a new square, and call the new square Square 2. The entries of Square 2 corresponding to entries \( a, b, \) and \( c \) of Square 1 are then \( a + n, b + n, \) and \( c + n \), respectively.

\[
\begin{array}{c c c}
\text{Square 2} \\
| a + n | & | b + n | \\
<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>b + n</td>
<td>&amp;</td>
</tr>
</tbody>
</table>
\end{array}
\]

The sum of the entries in this row, column, or diagonal is then

\[
(a + n) + (b + n) + (c + n).
\]

Rearranging this sum using the commutative and associative properties of addition, we obtain

\[
(a + b + c) + (n + n + n).
\]

Recalling that \( a + b + c = M \) and noting that \( n + n + n = 3n \), this sum becomes

\[
M + 3n.
\]

Now since this holds for any row, column, or diagonal, we have shown that the sum of the entries in each row, column, or diagonal of Square 2 is the same number, namely \( M + 3n \). It follows that Square 2 is a magic square with magic number \( M + 3n \) as claimed. \( \square \)