

Verifying Magic Square Properties Sample Proof

Theorem. Adding the same number n to each entry in a 3 by 3 magic square with magic number M yields a magic square with magic number $M + 3n$.

Proof. Suppose we are given a 3 by 3 magic square, called Square 1, and the three numbers in some row, column, or diagonal are represented by the variables a , b , and c .

Square 1

a		
	b	
		c

Suppose further that the magic number for Square 1 is represented by the variable M . It follows from the definition of a magic square that

$$a + b + c = M$$

and that this holds for *any* row, column, or diagonal with entries a , b , and c .

Now suppose we add the same number n to each entry in Square 1 to obtain a new square, and call the new square Square 2. The entries of Square 2 corresponding to entries a , b , and c of Square 1 are then $a + n$, $b + n$, and $c + n$, respectively.

Square 2

$a + n$		
	$b + n$	
		$c + n$

The *sum* of the entries in this row, column, or diagonal is then

$$(a + n) + (b + n) + (c + n).$$

Rearranging this sum using the commutative and associative properties of addition, we obtain

$$(a + b + c) + (n + n + n).$$

Recalling that $a + b + c = M$ and noting that $n + n + n = 3n$, this sum becomes

$$M + 3n.$$

Now since this holds for *any* row, column, or diagonal, we have shown that the sum of the entries in each row, column, or diagonal of Square 2 is the same number, namely $M + 3n$. It follows that Square 2 is a magic square with magic number $M + 3n$ as claimed. \square