(10)

Lec 4
1/30/09

(EXAMPLES, cont.)

③ $V = \mathbb{R}^3$, $W = \{(a,b,c) \in \mathbb{R}^3 \mid a,b,c \in \mathbb{Z}\}$ is not a subspace of V.

It is true that W is nonempty and closed under vector addition. However, W is not closed under scalar multiplication:

$\frac{1}{2} \in \mathbb{R}$, $(1,0,0) \in W$, but $\frac{1}{2}(1,0,0) = (\frac{1}{2},0,0)$ is not in W. $\square$

Remarks: Since closures must hold for all vectors and scalars, it is sufficient to find either two vectors $\bar{v}, \bar{w} \in W$ with $\bar{v} + \bar{w} \notin W$ or a scalar α and vector $\bar{w} \in W$ with $\alpha \bar{w} \notin W$ in order to show W is not a subspace.

A general argument is not needed and usually does not work. For example, saying that "if $r \in \mathbb{R}$ and $(a,b,c) \in W$, then $r(a,b,c) = (ra, rb, rc)$, and ra, rb, rc are not integers" is not a legitimate argument. There are $r \in \mathbb{R}$, $(a,b,c) \in W$ with $r(a,b,c) \in W$.

④ $V = M_n(\mathbb{R})$, $W = \{A \in M_n(\mathbb{R}) \mid A^T = A\}$ = Symmetric matrices. is a subspace of V.

[Review §2.6 for properties of the transpose.]

(i) Since $\bar{O}^T = \bar{O}$ ($\bar{O}$ = $n \times n$ zero matrix), $\bar{O} \in W$ and $W \neq \emptyset$.

(ii) If $A, B \in W$, so $A^T = A$, $B^T = B$, then $(A+B)^T = A^T + B^T = A + B$ by Theorem 2.3 (i), so $A + B \in W$.

(iii) if $\alpha \in \mathbb{R}$, $A \in W$, so $A^T = A$, then $(\alpha A)^T = \alpha (A^T) = \alpha A$ by Theorem 2.3 (iii), so $\alpha A \in W$.

Since (i), (ii), (iii) hold, W is a subspace of V.

⑤ If U, W are subspaces of V then $U \cap W$ is a subspace of V [See Theorem 4.3 in text.]

Note that $U \cup W$ is generally not a subspace of V.