We can generalize this Theorem to any direct sum of invariant subspaces.

Theorem: Let T be a linear operator on V and let $V = W_1 \oplus \cdots \oplus W_r$, where $W_1, \ldots, W_r$ are T invariant subspaces of V. Let $T_i = T_{W_i}$, the restriction of T to W_i, for each i.
Let $\Delta_T(t)$ be the characteristic polynomial of T,
$\Delta_i(t)$ the characteristic polynomial of T_i,
$m_T(t)$ the minimal polynomial of T,
$m_i(t)$ the minimal polynomial of T_i.
If S_i is a basis for W_i, so that $S = S_1 \cup \cdots \cup S_r$ is a basis for V, then

(i) The matrix of T relative to S is block diagonal:

$$A = [T]_S = \begin{bmatrix} A_1 & & & O \\ & A_2 & & \\ & & \ddots & \\ O & & & A_r \end{bmatrix}, \text{ where } A_i = [T_i]_{S_i}.$$

(ii) $\Delta_T(t) = \Delta_1(t)\Delta_2(t)\cdots\Delta_r(t)$
(in particular, $\Delta_i(t)$ is a factor of $\Delta_T(t)$ for all i)

(iii) $m_T(t) = \text{lcm}\{m_1(t), m_2(t), \ldots, m_r(t)\}$
(in particular, $m_i(t)$ is a factor of $m_T(t)$ for all i).

We write $T = T_1 \oplus T_2 \oplus \cdots \oplus T_r$ and $A = [T]_S = A_1 \oplus A_2 \oplus \cdots \oplus A_r$.
□

This theorem says we can work on one invariant subspace at a time, and find a "good" basis for W_i.

End Lec 33

END EXAM III