Lemma 2: Let T be a linear operator on V with minimal polynomial $m(t)$. If $m(t) = g(t)h(t)$ with $g(t), h(t)$ monic and relatively prime, and $U = \ker g(T)$, $W = \ker h(T)$, then
$g(t)$ is the minimal polynomial of the restriction T_U of T to U, and $h(t)$ is the minimal polynomial of the restriction T_W of T to W.

Proof: Since $m(T) = 0$, we have $V = U \oplus W$, with U, W both T-invariant, by Lemma 1. Hence we know $m(t) = \mathrm{lcm}\{m_{T_U}(t), m_{T_W}(t)\}$, where m_{T_U}, m_{T_W} is the minimal polynomial of T_U, T_W respectively.

Since $U = \ker g(T)$ and $W = \ker h(T)$, we have $g(T_U) = (g(T))_U = 0$ and $h(T_W) = (h(T))_W = 0$, hence
$m_{T_U}(t)$ is a factor of $g(t)$ and $m_{T_W}(t)$ is a factor of $h(t)$.

Since $g(t), h(t)$ are relatively prime, their factors m_{T_U}, m_{T_W} have no common factors either, and so
$$m(t) = \mathrm{lcm}\{m_{T_U}(t), m_{T_W}(t)\} = m_{T_U}(t)\, m_{T_W}(t) \text{ [Polynomials, \#8]}.$$
By hypothesis, $m(t) = g(t)h(t)$. Therefore, since $m_{T_U}(t)$ is a factor of $g(t)$ and $m_{T_W}(t)$ is a factor of $h(t)$, and all of these polynomials are monic, we have $m_{T_U}(t) = g(t)$ and $m_{T_W}(t) = h(t)$, as claimed. $\square$

Theorem (Primary Decomposition) Let T be a linear operator on V with minimal polynomial $m(t)$. If
$$m(t) = f_1(t)^{l_1} f_2(t)^{l_2} \cdots f_r(t)^{l_r},$$
where the $f_i(t)$ are distinct, monic, irreducible polynomials, and $W_i = \ker f_i(T)^{l_i}$ for each i, then

W_i is T-invariant for each i,

$V = W_1 \oplus W_2 \oplus \cdots \oplus W_r$, and

$m_i(t) = f_i(t)^{l_i}$ is the minimal polynomial of the restriction $T_i = T_{W_i}$ of T to W_i.

Proof: Use induction and Lemmas 1, 2 and [Polynomials, #5].
[See Problem 10.11]. $\square$