The following theorem gives all of the information required to determine the Jordan Form for any matrix or operator:

Theorem: [See Handout] Let T be a linear operator on V with $\Delta_T(t) = (t-\lambda_1)^{n_1}(t-\lambda_2)^{n_2}\cdots(t-\lambda_r)^{n_r}$, $m_T(t) = (t-\lambda_1)^{l_1}(t-\lambda_2)^{l_2}\cdots(t-\lambda_r)^{l_r}$, where the λ_i are distinct eigenvalues. The Jordan Canonical Form satisfies the following for each eigenvalue λ_i of T:

(1) The number of λ_i on the diagonal is n_i, the algebraic multiplicity of λ_i; in particular, the sum of the sizes of the λ_i-blocks is n_i.

(2) The size of the largest λ_i-block is l_i; that is, there is at least one λ_i-block of size l_i and every λ_i-block is of size $\leq l_i$.

(3) The number of λ_i-blocks is equal to $\dim E_{\lambda_i} = \dim \ker(T-\lambda_i I) =$ geometric multiplicity of λ_i.

(4) The number of λ_i-blocks of size k is equal to

$$b_k = 2\dim\ker(T-\lambda_i I)^k - \dim\ker(T-\lambda_i I)^{k-1} - \dim\ker(T-\lambda_i I)^{k+1}$$

$$= 2\,\text{nullity}(T-\lambda_i I)^k - \text{nullity}(T-\lambda_i I)^{k-1} - \text{nullity}(T-\lambda_i I)^{k+1}$$

$$= \text{rank}(T-\lambda_i I)^{k-1} - 2\,\text{rank}(T-\lambda_i I)^k + \text{rank}(T-\lambda_i I)^{k+1}.$$

Notes on Proof:

Statement (1) follows from the fact that the Jordan form is triangular with the eigenvalues on the diagonal.

[Continued →]