Lec 38, 5/4/09

EXAMPLES:

① The possible Jordan forms for a 2×2 matrix are:

$\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$, $\Delta(t)=(t-\lambda)^2$, $m(t)=t-\lambda$

$\begin{bmatrix}\lambda & 1\\ 0 & \lambda\end{bmatrix}$, $\Delta(t)=(t-\lambda)^2$, $m(t)=(t-\lambda)^2$

$\begin{bmatrix}\lambda & 0\\ 0 & \mu\end{bmatrix}$, $\mu\neq\lambda$, $\Delta(t)=(t-\lambda)(t-\mu)=m(t)$.

In all cases, the JCF is determined by $\Delta(t)$ and $m(t)$.

② A 3×3 matrix can have 1, 2, or 3 distinct eigenvalues.

Case 1: if 3 distinct eigenvalues λ,μ,η, then
$m(t)=\Delta(t)=(t-\lambda)(t-\mu)(t-\eta)$ and JCF is $\begin{bmatrix}\lambda&0&0\\0&\mu&0\\0&0&\eta\end{bmatrix}$.

Case 2: If 2 distinct eigenvalues, λ,μ, we may assume λ has algebraic multiplicity 2, $\Delta(t)=(t-\lambda)^2(t-\mu)$
There are two possible minimal polynomials, with JCF:

$m(t)=(t-\lambda)(t-\mu)$ $\quad\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\mu\end{bmatrix}$

$m(t)=(t-\lambda)^2(t-\mu)$ $\quad\begin{bmatrix}\lambda&1&0\\0&\lambda&0\\0&0&\mu\end{bmatrix}$

Case 3 if 1 eigenvalue λ, then $\Delta(t)=(t-\lambda)^3$.
There are three possibilities for $m(t)$, with JCF:

$m(t)=(t-\lambda)$ $\quad\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}$

$m(t)=(t-\lambda)^2$ $\quad\begin{bmatrix}\lambda&1&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}$

$m(t)=(t-\lambda)^3$ $\quad\begin{bmatrix}\lambda&1&0\\0&\lambda&1\\0&0&\lambda\end{bmatrix}$

Note: As seen in Example ② on page 109, there are two possible Jordan forms for a 4×4 matrix with $\Delta(t)=(t-\lambda)^4$, $m(t)=(t-\lambda)^2$.

Conversely, given the Jordan Form, we can read off the multiplicity of $t-\lambda$ in $\Delta(t)$ (number of λ on diagonal) and in $m(t)$ (size of largest λ-block) for each eigenvalue λ.