[EXAMPLES, cont]

(3) Find all possible Jordan Forms for an operator with
$\Delta(t) = (t-2)^4 (t+1)^2$ and $m(t) = (t-2)^3 (t+1)$.

From $\Delta(t)$, we know there are four 2's, two -1's on the diagonal.
There is a 2-block of size 3, hence one of size 1, by $m(t)$.
there are only -1-blocks of size 1 by $m(t)$. The only JCF is

$$J = \begin{bmatrix} 2 & 1 & & & & \\ & 2 & 1 & & & \\ & & 2 & & & \\ & & & 2 & & \\ & & & & -1 & \\ & & & & & -1 \end{bmatrix}.$$

[All missing entries are 0.]

(4) Find all possible Jordan Forms for an operator with
$\Delta(t) = (t-1)^5 (t-2)^4$, $m(t) = (t-1)^3 (t-2)^2$.

For $\lambda = 1$, there is a block of size 3 and five 1's on the diagonal.
Hence the possible sizes of 1-blocks are 3,2 or 3,1,1.
For $\lambda = 2$, there is a block of size 2 and four 2's on the diagonal.
The possible block sizes for $\lambda = 2$ are 2,2 or 2,1,1.
Hence there are 4 possible Jordan Forms.

Note that for each eigenvalue, the two cases are distinguished by the number of blocks, hence knowing the dimensions of E_1 and E_2 would determine the JCF.

For example, if $\dim E_1 = 3$ and $\dim E_2 = 2$, the JCF is

$$\begin{bmatrix} 1 & 1 & & & & & & & \\ & 1 & 1 & & & & & & \\ & & 1 & & & & & & \\ & & & 1 & & & & & \\ & & & & 1 & & & & \\ & & & & & 2 & 1 & & \\ & & & & & & 2 & & \\ & & & & & & & 2 & 1 \\ & & & & & & & & 2 \end{bmatrix}.$$