[EXAMPLES, cont.]

⑤ Find the possible Jordan Forms for A if $\Delta_A(t) = (t-2)^8$ and $m_A(t) = (t-2)^4$, given the following information:

(a) Given only $\Delta_A(t)$, $m_A(t)$.

We know A is an 8×8 matrix with eight 2's on the diagonal. There is a block of size 4, and the sum of the block sizes is 8.

The possible Jordan block sizes are:

4, 4
4, 3, 1
4, 2, 2
4, 2, 1, 1
4, 1, 1, 1, 1.

(b) Given $\Delta_A(t)$, $m_A(t)$, and $\dim E_2 = 3$.

In this case, we know that there are 3 blocks for $\lambda = 2$, so the block sizes are either

4, 3, 1 or
4, 2, 2.

In this case, Parts 1-3 of the theorem are not sufficient to determine the JCF. We would need to know the number of blocks of size 1, 2, or 3.

(c) Given $\Delta_A(t)$, $m_A(t)$, $\dim E_2 = 3$, and $\dim N(A-2I)^2 = 5$.

As above, the block sizes are 4, 3, 1 or 4, 2, 2.

By the theorem, the number of blocks of size 1 is

$$b_1 = -\dim N(A-2I)^0 + 2\dim N(A-2I)^1 - \dim N(A-2I)^2$$
$$= 0 + 2(3) - 5 = 1.$$

Since there is one block of size 1, the block sizes must be 4, 3, 1.

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