

[EXAMPLES, CONT.]

⑧ Find the Jordan Canonical form of

$$C = \begin{bmatrix} 2 & 0 & 2 & 3 \\ 4 & 3 & -8 & -12 \\ -2 & 0 & 7 & 6 \\ 1 & 0 & -2 & 0 \end{bmatrix}, \text{ with } \Delta_C(t) = (t-3)^4.$$

[check - expand  $\det(tI - C)$  along the 2<sup>nd</sup> column.]

$$\text{we have } C - 3I = \begin{bmatrix} -1 & 0 & 2 & 3 \\ 4 & 0 & -8 & -12 \\ -2 & 0 & 4 & 6 \\ 1 & 0 & -2 & -3 \end{bmatrix} \text{ and } (C - 3I)^2 = 0,$$

$$\text{hence } \underline{m_C(t) = (t-3)^2}.$$

We know the Jordan Form has four 3's on the diagonal, and the largest block is of size 2. Hence the block sizes are either 2, 2 or 2, 1, 1. The two forms are distinguished by the number of blocks,  $\dim E_2$ .

$$C - 3I \rightarrow \begin{bmatrix} 1 & 0 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ so } \dim \mathcal{N}(C - 3I) = 3,$$

hence there are three blocks and the sizes are 2, 1, 1 and the Jordan Form is

$$\begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ \hline 0 & 0 & 3 & 0 \\ \hline 0 & 0 & 0 & 3 \end{bmatrix}.$$

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