(18)

EXAMPLES:

① If $S=\{\bar{v}_1,\dots,\bar{v}_n\}$ is linearly independent and $\beta_1,\dots,\beta_n$ are nonzero scalars, then $S'=\{\beta_1\bar{v}_1,\dots,\beta_n\bar{v}_n\}$ is linearly independent.

Proof: Suppose $\alpha_1(\beta_1\bar{v}_1)+\cdots+\alpha_n(\beta_n\bar{v}_n)=\bar{0}$. By axiom [M3], we have $(\alpha_1\beta_1)\bar{v}_1+\cdots+(\alpha_n\beta_n)\bar{v}_n=\bar{0}$. Since $\{\bar{v}_1,\dots,\bar{v}_n\}$ is linearly independent, this implies $\alpha_1\beta_1=\alpha_2\beta_2=\cdots=\alpha_n\beta_n=0$. Since $\beta_i\neq 0$ and $\alpha_i\beta_i=0$ for all i, this implies $\alpha_i=0$ for all i. Hence S' is linearly independent. $\square$

In order to determine whether a set $S=\{\bar{v}_1,\dots,\bar{v}_n\}$ is linearly independent, we need to determine if the equation $\alpha_1\bar{v}_1+\cdots+\alpha_n\bar{v}_n=\bar{0}$ (*) has a unique solution $\alpha_1=\alpha_2=\cdots=\alpha_n=0$.

In practice, equation (*) is translated into a (homogeneous) system of linear equations. Such a system has a unique solution if and only if there are no free variables in the echelon form, that is, there is a pivot in every column of the echelon form of the coefficient matrix.

[See handouts on Echelon Form (uniqueness of solutions) and Spanning & Linear Independence (linear independence). See also Ch. 3 of text, especially §§ 3.7–3.11.]

② Determine whether the set

$$S=\{(2,-1,2),\ (1,2,-1),\ (4,3,0)\}\subseteq\mathbb{R}^3$$

is linearly independent or linearly dependent.

We need to determine whether the equation

$$x(2,-1,2)+y(1,2,-1)+z(4,3,0)=(0,0,0)$$

End Lec6

has only the solution $x=y=z=0$.