where $\alpha_i, \beta_i \in K$ and $\bar{v}_i \in S$. By the vector space axioms, we can write $(\alpha_1 - \beta_1)\bar{v}_1 + \cdots + (\alpha_n - \beta_n)\bar{v}_n = \bar{0}$. By linear independence of S, $\alpha_i - \beta_i = 0$, that is, $\alpha_i = \beta_i$ for all i. Hence the expression is unique. □

EXAMPLES (standard bases)

① $\mathbb{R}_n$: $B = \{(1,0,0,\dots,0), (0,1,0,\dots,0), (0,0,1,\dots,0), \dots (0,0,0,\dots,1)\}$ (n vectors).

② $M_{m,n}(\mathbb{R})$: $B = \{e_{ij} \mid i=1,\dots,m;\ j=1,\dots,n\}$, where e_{ij} is the matrix with 1 in the (i,j)-position and 0 elsewhere. ($m \cdot n$ vectors).

③ $P_n(t)$: $B = \{1, t, t^2, \dots, t^n\}$ ($n+1$ vectors).

④ $P(t)$: $B = \{1, t, t^2, \dots\}$ (infinitely many vectors).

Theorem Any two bases for a vector space V have the same number of elements.

Proof: See Lemma 4.13, theorem 4.12, proved in Problems 4.35, 4.36. □

Defn: The dimension of V, denoted $\dim_K V$ or $\dim V$, is the number of vectors in a basis for V.

Examples: Given the standard bases above, we have:

① $\dim \mathbb{R}^n = n$
② $\dim M_{m,n}(\mathbb{R}) = m \cdot n$
③ $\dim P_n(t) = n+1$
④ $\dim P(t)$ is infinite.

End Lec 7